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FRACTIONS &c.

FIFTY STEPS

IN

MEASURES, MULTIPLES, VULGAR & DECIMAL FRACTIONS:

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PREFACE.

It is with a hope of giving some encouragement to *Self-Teaching*, at a comparatively early stage, that this Book on Fractions has been prepared.

It is not pretended that such *Self-Teaching* would not generally need to be supplemented by *viva voce* instruction and examination. But, surely, what has been learnt, has been learnt to little purpose, if, at the age of fifteen or sixteen, pupils are unable to make the least advance *by themselves*, but are, at every new step, absolutely dependent on the guidance of a Teacher.

IT is suggested that Pupils should

invent for themselves,

work out,

and, when possible, *prove*

examples similar to those given in the Exercises.

MEASURES OR FACTORS AND MULTIPLES.

1. Factors and Simple Factors.

The factors of 12 are 2, 3, 4, 6,

Because each of these numbers divides 12 *exactly*, that is without a remainder.

Strictly speaking 1 and 12 are also factors of 12; but we will not take them into consideration now.

Of the four factors 2, 3, 4, 6,

2 and 3 are *simple* or *prime* factors;

Because neither of them can be divided exactly by any other number.

4 and 6 are not simple factors;

Because each of them can be exactly divided by some other number, *viz.* 4 by 2, and 6 by 2 and by 3.

Ex. 1.—Write down all the factors of each of the following numbers, and say which are *simple* factors,

10, 18, 20, 28, 36, 42.

Specimen Example.—What are the factors of 42? and which of them are simple factors?

42 can be exactly divided by 2, 3, 6, 7, 14, and 21;

These therefore are the factors of 42.

Of the factors of 42, the only ones which cannot be divided exactly by some other number, are 2, 3, 7.

These therefore are the simple factors of 42.

It should be clearly understood what is meant by such a *continued product* as $2 \times 3 \times 4 \times 5$;

$$\begin{aligned} & 2 \times 3 \times 4 \times 5 \\ &= 2 \times 3 \times 20 \ldots\text{ because } 4 \times 5 = 20 \\ &= 2 \times 60 \ldots\text{ because } 3 \times 20 = 60 \\ &= 120 \end{aligned}$$

2. Resolve 12 into factors.

This does not mean

"Write down all the factors of 12."

But "Write down only factors which when multiplied together will exactly make up 12."

We write down then

2×6, because $2 \times 6 = 12$.

or 3×4, because $3 \times 4 = 12$.

or $3 \times 2 \times 2$, because $3 \times 2 \times 2 = 12$.

Ex. 2.—Resolve into factors

20, 26, 28, 36, 42.

Specimen Example.— Resolve 42 into factors.

We know that $7 \times 6 = 42$;

also $2 \times 21 = 42$;

also $3 \times 14 = 42$;

also $3 \times 2 \times 7 = 42$.

Ans. 7×6; *or* 2×21; *or* 3×14; *or* $3 \times 2 \times 7$.

3. Resolve 12 into simple factors.

This means

"Resolve 12 into factors in such a way that each factor is a *simple factor* or *prime number*."

We saw that 12 might be broken up into factors by taking 2 × 6.

But 2 × 6 will not do here, for 6 is not a simple factor.

Nor will 3 × 4; for 4 is not a simple factor.

We must take 2 × 2 × 3;

For each of these is a simple factor.

Ans. 2 × 2 × 3.

Ex. 3.—Resolve into their simple factors

20, 24, 28, 32, 56, 60, 75, 80, 88, 90, 100.

Specimen Example.—Resolve into simple factors 100 & 56.

$100 = 10 \times 10$ (10 however is not a simple factor.)

$= 5 \times 2 \times 5 \times 2$ (here each number is a simple factor.)

Ans. 5 × 5 × 2 × 2.

$56 = 7 \times 8$.

Here 7 is a simple factor, 8 is not; $8 = 2 \times 4, = 2 \times 2 \times 2$.

We write down then

$$56 = 7 \times 8.$$
$$= 7 \times 2 \times 4.$$
$$= 7 \times 2 \times 2 \times 2.$$

Ans. 2 × 2 × 2 × 7.

Observe: It does not matter in what order we write the factors.

4. We have just found that

$$56 = 2 \times 2 \times 2 \times 7 \text{ and } 100 = 2 \times 2 \times 5 \times 5.$$

We may write, as abbreviations :—

2^2 for 2×2; 5^2 for 5×5; 2^3 for $2 \times 2 \times 2$.

So that $100=2^2\times5^2$; $56=2^3\times7$.

Observe: The little number, or index, up in the right hand corner, shews the number of 2's or of 5's that are to be multiplied together.

$3\times3\times3\times3\times7\times7$ may thus be expressed as $3^4\times7^2$.

And $13\times13\times13\times13\times13$ as 13^5, &c.

7^2 is read as "7 squared," and means 7×7 or 49;

2^3 is read as "2 cubed," and means $2\times2\times2$ or 8;

3^4 is read as "3 to the fourth," and means $3\times3\times3\times3$ or 81; and so in other cases.

Warning: It is a common mistake to say, for instance, that 7^2 means "2×7" or 14.

Ex. 4. A.—Express in the manner just explained:

$3\times3\times2$; $3\times3\times3\times2\times2$; $5\times5\times5\times11\times11\times7$.

also 9, 8, 16, 18, 20, 24.

Observe: The last five numbers must *first* be broken up into simple factors, *then* expressed in the abbreviated form

thus, $24=2\times2\times2\times3=2^3\times3$.

B.—What numbers are expressed by

2^2, 4^2, 3^3, 4^3, 2^4, 3^4, 7^3, 5^4, 9^2;

$2^2\times3^3$, $3^2\times2^3$, $9^2\times2^3$?

5. Let us take any number, for instance 60;

60 expressed in simple factors is

$$2\times2\times3\times5.$$

Since 60 *is* $2\times2\times3\times5$, we can *see* that 60 *contains* 2;

For we see a 2 among the factors of 60.

60 then can be exactly divided by 2.

And when 60 is divided by 2 what is the quotient? We can *see* this also:—

Remove 2 from the row of factors:

$$2 \times 2 \times 3 \times 5;$$

We have left $2 \times 3 \times 5$ or 30.

30 then is the quotient when 60 is divided by 2.

What is the quotient when 60 is divided by 3? *Ans.* $2 \times 2 \times 5$.

We can *see* also that 60 can be exactly divided by 2×3 or 6;

For we *see* a 2×3 contained in $2 \times 2 \times 3 \times 5$.

And we see what the quotient is when 60 is divided by 2×3, *viz.*, 2×5 or 10.

For when from $2 \times 2 \times 3 \times 5$ we remove 2×3

We have left 2×5.

Similarly when 60 is divided by $2 \times 2 \times 3$ or 12, the quotient is 5.

Warning: This way of removing factors has nothing whatever to do with subtraction.

Ex. 5.—Write down the several quotients

(i) when $2 \times 3 \times 3 \times 5$ is divided by

$2, 3, 5, 2 \times 3, 3 \times 3, 2 \times 3 \times 3, 3 \times 3 \times 5$.

(ii) when 72 is divided by

$2, 3, 2 \times 2, 2 \times 2 \times 2, 2 \times 2 \times 3, 2 \times 2 \times 3 \times 3$.

(Here 72 must first be expressed in simple factors.)

(iii) when $2^3 \times 3^4 \times 5$ is divided by

$2^2, 3, 2 \times 3, 3^4 \times 5, 2^2 \times 3^3, 2 \times 5$.

(Here each number should be first written out in the *un*abbreviated form, thus, $2 \times 2 \times 2$ for 2^3.)

6. When we have to divide one number by another, we may, if we can, *cancel* before dividing.

Ex. Divide 8000 by 300.

We see at once that both 8000 and 300 can be exactly divided by the same number, 100.

And so we may divide them both by 100 before we divide one by the other. That is, we may *cancel* 100.

Cancelling then means, "dividing each number by some common factor."

We may cancel *only* when one number has to be *divided* by the other.

To return to the example:

Dividing 8000 and 300 each by 100,

We have 80 and 3.

Now dividing 80 by 3,

We have $3\overline{)80}$

26, and 2 over.

This remainder "2," however, *must be multiplied by the* 100 *which we cancelled.*

For 200, not 2, is the true remainder.

To see the reason for this,

Suppose that 8000 soldiers have to be divided into regiments of 300 each;

If a Company consist of 100 soldiers, we may say that 80 Companies have to be divided into regiments of 3 Companies each.

Thus we exchange the numbers 8000 and 300 for 80 and 3.

Now when we divide 80 by 3, the remainder 2 will represent 2 *Companies* or 200 *men.*

Thus the remainder has to be multiplied by the common factor which was cancelled.

A similar explanation applies to such an example as the following :—

Divide 12315 things into 30 equal sets,

$$\begin{array}{r} 5)\underline{12315} \\ 6)\underline{2463} \\ 410 \end{array} \text{ and 3 over.}$$

Common Rule.—Multiply the remainder 3, by the first divisor 5, and we have the true remainder, 15.

Explanation.—By first dividing by 5 we bring the 12315 things to 2463 sets of 5 each.

And the 3 that is over in the next line represents 3 of these *sets of* 5 *things.*

So to get the true remainder we multiply the 3 by 5.

Ans. 410 and 15 *over.*

Ex. 6.—Divide 11050 by 150 (cancel 50.)
880 by 160 (cancel 80.)
99099 by 66 (cancel 33.)
10000 by 300, and by 30.
17017 by 170.

Specimen Example :—

Divide 9990999 by 444.

Here 111 may be cancelled.

$9990999 \div 111 = 90009$, and $444 \div 111 = 4$.

$$\begin{array}{r} 4)\underline{90009} \\ 22502, \end{array} \text{ and 1 over.}$$

This 1 represents 1 set of 111 units, or 111 units.

Answer 22502 and 111 over.

QUESTIONS ON STEPS 1–6.

1. What is a factor of a number?

What is a simple, or prime, factor?

Write down all the factors of 28, saying which are prime factors.

What is the value of $4 \times 5 \times 6 \times 7$?

2. What is meant by *resolving a number into factors?*

Resolve 28 into factors in all possible ways.

3. Resolve into simple factors 96 and 960.

State what is meant by resolving into *simple* factors.

4. For what do the following stand

$$3^5, \quad 5^3, \quad 4^4?$$

What numbers are represented by

$$2^2 \times 3, \quad 2^3 \times 3^4, \quad 4^2 \times 5^3 \times 7?$$

5. Divide $3 \times 4 \times 5 \times 6$ by 3×5.

Divide $3 \times 4 \times 5 \times 7 \times 11$ by 21, 55, and 660.

6. When *only* may we cancel between two numbers?

Divide (cancelling first)

10000 by 350 and by 30.
1008 by 144 and by 180.

What must be remembered about the remainders? Give the reason for it.

7. **A Measure** is the same thing as a Factor.

Thus 6 is a measure of 18.

Instead of saying

"6 divides 18 exactly,"

We may say

"6 measures 18."

A Number is exactly divisible, or is measured

By 10, if it ends with 0.

By 5, if it ends with 0 or 5.

By 2, if 2 is a measure of the number expressed by its last digit, *i. e.* if the number ends with 0, 2, 4, 6, or 8.

By 4, if 4 is a measure of the number expressed by its last two digits.

Thus 4 measures 10116 since it measures 16.

By 8, if 8 is a measure of the number expresssed by its last three digits.

Thus 8 measures 11648 since it measures 648.

By 3, if the sum of its digits is measured by 3.

Thus 27138 is measured by 3, for when we add together 2, 7, 1, 3, and 8, the number so found, 21, is measured by 3.

By 9, if the sum of its digits is measured by 9.

Thus 12345678 is measured by 9, for when we add together 1, 2, 3, 4, 5, 6, 7, 8, 9, the number so found, 36, is measured by 9.

By 11, when, if we add together the 1st, 3rd, 5th, &c. digits
and also the 2nd, 4th, 6th, &c. digits,
the difference between the two sums is 0, or is itself measured by 11.

Thus 219283746 is measured by 11.

For adding 2, 9, 8, 7, 6, we have 32.

And adding 1, 2, 3, 4, we have 10.

And the difference between 32 and 10—being 22—is measured by 11.

Ex. 7.—What measures or factors, less than 12, has each of the following numbers—

1128, 7128, 3131, 1536, 10000?

Specimen Example.—7128.

The last digit is 8; 2 measures 8; 2 is a measure of 7128.

The last two digits are 28; 4 measures 28; 4 is a measure of 7128.

The last three digits are 128; 8 measures 128; 8 is a measure of 7128.

$7+1+2+8=18$; and both 3 and 9 are measures of 18; therefore both 3 and 9 are measures of 7128.

We see at once that neither 5 nor 10 are measures.

And we may find by trial that 7 is not.

And as for 11,

$$7+2=9 \text{ and } 1+8=9.$$

And the difference between 9 and 9 is 0.

Therefore 11 is a measure.

And because 2 and 3 are both measures of 7128, therefore 2×3, or 6, is also a measure.

Ans. 2, 3, 4, 6, 8, 9, 11.

8. To resolve, when practicable, large numbers into their simple Factors.

Example.—137445.

Find by the preceding methods what measures there are of this number, less than 12.

We obtain 5, and 3, and 11.

Observe: $1+7+4=12$, and $3+4+5=12$, hence 11 is a measure.

We accordingly divide by these numbers successively.

We divide too by 7, because we find on trial that 7 is also a measure;

We then examine whether we can divide by any of these same numbers *a second time.*

We shall be able to divide again by 7, but by 7 only.

We have accordingly

$$\begin{array}{r} 5\underline{)137445} \\ 3\underline{)27489} \\ 11\underline{)9163} \\ 7\underline{)833} \\ 7\underline{)119} \\ 17 \end{array}$$

Ans. $5 \times 3 \times 11 \times 7 \times 7 \times 17$.

Any measures which are not prime numbers should of course be resolved into their simple factors in the result.

Ex. 8.—Resolve into simple factors

15015, 3570, 1584, 1287,
4000, 1029, 1870, 6237.

9. Common Measures.

3 is a measure or factor of 12, and of 27, and of 30.

In other words

3 is a Common Measure of 12, 27, and 30.

Ex. 9.—Write down all the Common Measures of 20 and 24; of 50 and 60; of 75 and 120; of 8, 12, and 20.

Specimen Example.—Find all the Common Measures of 84 and 238.

Expressing them in simple factors, we have

$$84=7\times 12, \qquad 238=2\times 119,$$
$$=7\times 3\times 2\times 2, \qquad =2\times 7\times 17.$$

The factors that occur in both, *or the Common Measures,* are 7 and 2, and 7 × 2 or 14.

Ans. 7, 2, 14.

Observe.—The Common Measures of

$$2\times 3\times 5\times 7 \text{ and } 2\times 3\times 5\times 8$$

are not only 2, 3, 5, but also 3×5, 5×2, 2×3, $2\times 3\times 5$.

And of these $2\times 3\times 5$ is plainly the Greatest: thus,

10. The Greatest of the Common Measures of two, or more, numbers is found by multiplying together all the simple factors that are common to all the numbers.

Ex. 10. A.—Find the Greatest of the Common Measures of 24 and 30; 96 and 128; 56 and 84; 24, 30, and 96; 128, 56, and 84.

Specimen Example.—36 and 60.

$$36=6\times 6=2\times 3\times 2\times 3.$$
$$60=6\times 10=2\times 3\times 2\times 5.$$

The simple factors common to both are

2, 3, 2.

And the greatest of the Common Measures is

$2\times 3\times 2$, or 12.

As another example, take 36, 60, and 80.

As before $36=2\times 2\times 3\times 3.$

$60=2\times 2\times 3\times 5.$

And $80=2\times 2\times 2\times 2\times 5.$

The simple factors common to all three numbers are 2, 2.

$\therefore$ 2×2, or 4, is the G. C. M. required.

The examples in Ex. 10 should also be worked out by the Common Rule, that it may be seen that the results agree.

Specimen Example in the Common Rule of G. C. M.—

(The easiest way for large Numbers.)

Find the G. C. M. of 96 and 1070.

```
        96)1070(11
            96
           ---
           110
            96
            --
            14)96(6
               84
               --
               12)14(1
                  12
                  --
                   2)12(6
Ans. 2.              12
```

Ex. 10. B.—Find, both by the Common Rule and also by taking factors, the G. C. M. of each of the following pairs of numbers—

180, 405; 168, 490; 2233, 1218; 6930, 29997;
999, 12321; 4264, 90282.

11. A Multiple of a number is a number which is exactly divisible by the given number.

A Common Multiple of two or more given numbers is a number which can be exactly divided by each of the given numbers.

Example.— Write down all the numbers of which 24 is a Common Multiple.

Ans. 2, 3, 4, 6, 8, 12, 24; for 24 can be exactly divided by each of these.

Example.—Write down two common multiples of 4, 5, 6.

Ans. 60 and 120; for each of these numbers can be exactly divided by 4, by 5, and by 6.

We can always find a Common Multiple of any given numbers by multiplying them all together.

Ex. 11.—Write down two Common Multiples of 2 and 3; of 3 and 4; of 2 and 5; of 2, 3, and 4; of 3, 4, and 5.

Write down all the numbers of which 20 is a Common Multiple.

12. **To find the** *Least* **of the Common Multiples of several numbers**; that is, the least number which *contains* them all, or which can be divided by each without remainder.

Let the numbers be 6, 9, 10, 30.

Express each in simple factors.

$$6=2\times3.$$
$$9=3\times3.$$
$$10=2\times5.$$
$$30=2\times3\times5.$$

Now build up a new number out of these simple factors, *not taking the same factor more than once over, unless it occurs more than once in the same line;* thus:—

$$2\times3\times3\times5 \text{ or } 90. \quad \textit{Ans. } 90.$$

We have taken 2, *but only one* 2, because, though 2 occurs three times, *it does not occur more than once in the same line.*

We have taken two 3's because 3 × 3 occurs *in one line.*

We have taken also 5.

No factor that occurs at all is left out.

And now we can plainly *see* that the number we have built up

$$2 \times 3 \times 3 \times 5$$

really contains each of the numbers with which we started.

We SEE that it contains amongst its factors

2×3, or 6.
also 3×3 or 9.
also 5×2 or 10.
also $2 \times 3 \times 5$ or 30.

Thus it is exactly divisible by, or is a multiple of, each one.

And it is the least number that is so, for it contains no superfluous factor.

Observe: In working examples we may at once leave out of consideration any of the given numbers which are contained in any other.

In the last Example we might have left out 6 and 10 altogether, and the result would have been the same; for clearly any number that contains 30 contains also both 6 and 10.

Ex. 12.—Find the Least of the Common Multiples or L.C.M.

of 9, 8, 18, 24, 30;
6, 40, 42, 18, 16;
10, 96, 84, 14, 40;
65, 100, 125, 130, 156;
51, 85, 170, 150;
22, 40, 121;
28, 63, 36;
81, 54. 42;
108, 162, 160, 270.

Specimen Example.—18, 24, 30, 40, 56.

$$18=3\times6=3\times3\times2.$$
$$24=3\times8=3\times2\times2\times2.$$
$$30=3\times10=3\times5\times2.$$
$$40=2\times20=2\times2\times2\times5.$$
$$56=7\times8=7\times2\times2\times2.$$

First put down $3\times3\times2$ from the top line;

From the second line put in two more 2's, thus,

$$3\times3\times2\times2\times2;$$

From the third line put in 5, thus,

$$3\times3\times2\times2\times2\times5;$$

From the fourth line we have not to put in anything, as we have three 3's and a 5 already.

From the fifth line put in 7, thus,

$$3\times3\times2\times2\times2\times5\times7.$$
$$=2520$$

Ans. 2520.

We may make it a little shorter in this way,

$$18=2\times3^2,\ 24=2^3\times3,\ 30=2\times3\times5,\ 40=2^3\times5,$$
$$56=2^3\times7.$$

Here the highest power, as it is called, of 2 is 2^3.

The highest power of 3 is 3^2.

And we also have a 5 and a 7, (but no 5^2 nor 7^2 nor higher powers of these numbers.)

From these we have

Ans. $2^3\times3^2\times5\times7$, or 2520.

QUESTIONS ON STEPS 7–12.

7. When can a number be measured by 3, 4, 5, 8, 9, 11 ? Find which of these numbers measure 157840122.

8. Resolve into simple factors

100000, 363363, and 676.

9. Find all the common measures of

60, 72, and 80.

10. Explain the term "Greatest Common Measure" and find the Greatest Common Measure (G. C. M.) of

72, 100, and 120.

11. What is a Common Multiple of two or more numbers ?

Write down all the numbers of which 100 is a Common Multiple.

12. Find the Least Common Multiple (L. C. M.) of

16, 18, 20, 24,

And *shew* that your result *contains* each of these four numbers.

13. Vulgar Fractions—

We know that

$$20 \div 4$$

is a short way of expressing

"20 divided by 4."

There is another way in which the same may be expressed; that is by the *Fraction* $\frac{20}{4}$.

We understand then that the *fraction* $\frac{20}{4}$ means the same as "20 divided by 4," and so is equal to 5.

$\frac{30}{10}$ means the same as "30 divided by 10," that is 3.

$\frac{7}{7}$ means the same as "7 divided by 7," that is 1.

$\frac{8}{1}$ means the same as "8 divided by 1," that is 8.

Ex. 13. A.—To what are the following fractions equal—

$$\frac{40}{8},\ \frac{12}{3},\ \frac{15}{5},\ \frac{20}{5},\ \frac{36}{2},\ \frac{35}{5},\ \frac{7}{7},\ \frac{11}{11},\ \frac{7}{1},\ \frac{11}{1}\ ?$$

Specimen Example.—$\frac{40}{8} = 40 \div 8 = 5$. *Ans.* 5.

Ex. 13. B.—Express as fractions $120 \div 8$, $90 \div 15$, $170 \div 35$, $10000 \div 16$, $21 \div 1$, $21 \div 21$.

Specimen Example.—$10000 \div 16 = \frac{10000}{16}$. *Ans.* $\frac{10000}{16}$.

14. Whole numbers may be expressed as Fractions.

We may, for example, turn 11 into any of the following fractions:—

$\frac{11}{1}$, $\frac{22}{2}$, $\frac{33}{3}$, &c.

Because $\frac{11}{1}$ *means* $11 \div 1$, that is 11.

$\frac{22}{2}$ *means* $22 \div 2$, that is 11.

$\frac{33}{3}$ *means* $33 \div 3$, that is 11.

Example.—Express 11 as a fraction whose lower figure is 7.

Ans. $\frac{77}{7}$.

Because $\frac{77}{7}$ *means* 77÷7, that is 11.

Ex. 14.—(1.) Express 11 as a fraction whose lower figure is 8.

(2.) Express 10 as a fraction whose lower figure is 5.

(3.) Write down any three fractions each equal to 9.

(4.) Write down any three fractions each equal to 13.

(5.) Express 14, 15, 16, each by different fractions whose lower figures are respectively 1, 4, 7, 3, 10, 20.

Specimen Example.—Express 18 by different fractions whose lower figures are respectively 1, 4, and 7.

$$18=\frac{18}{1}$$

$$18=\frac{18\times4}{4}=\frac{72}{4}$$

$$18=\frac{18\times7}{7}=\frac{126}{7}$$

Ans. $\frac{18}{1}$, $\frac{72}{4}$, $\frac{126}{7}$.

Observe: These results may be proved by division, thus,

$$\frac{18}{1}=18\div1=18.$$

$$\frac{72}{4}=72\div4=18.$$

$$\frac{126}{7}=126\div7=18.$$

15. In a fraction the upper figure is called the *Numerator,* **and the lower the** *Denominator.*

Thus in the fraction $\frac{8}{4}$

8 is the Numerator, 4 the Denominator.

All the fractions which we have had as yet,

As for instance $\frac{40}{8}$, $\frac{18}{1}$, $\frac{126}{7}$,

Are *improper* fractions, that is, fractions *whose Numerators are greater than their Denominators.*

We may have fractions *whose Numerators are less than their Denominators,* as $\frac{7}{12}$; these are called *proper* fractions.

Observe: $\frac{7}{12}$ *represents* "7 divided by 12;" but as 7 is *less* than 12 this explanation is not complete.

A further account of what we mean by a fraction will be given later.

16. Reducing to Lower Terms—

It is a general rule that

When one number has to be divided by another

we may, if possible, *cancel* before dividing;

That is, we may divide both by some *common measure* or number which will divide each exactly.

$\frac{35}{10}$ represents "35 divided by 10."

Here then we may, if possible, *cancel.*

5 is a common measure of 35 and 10.

We therefore divide each of them by 5; thus,

$$5)\ \frac{35}{10} = \frac{7}{2}.$$

This is called "reducing $\frac{35}{10}$ to Lower Terms."

Ex. 16.—Reduce to Lower Terms

$\frac{25}{10}$, $\frac{45}{10}$, $\frac{35}{14}$, $\frac{99}{22}$, $\frac{15}{65}$, $\frac{56}{63}$, $\frac{63}{72}$.

Specimen Example.—$\frac{63}{72}$; both 63 and 72 are exactly divisible by 9, thus we have

$$9)\ \frac{63}{72} = \frac{7}{8}. \quad \textit{Ans.}\ \frac{7}{8}.$$

17. To reduce a fraction to its Lowest Terms.

General Rule.—Cancel the G. C. M. of the Numerator and Denominator.

1st Way.—Resolve both Numerator and Denominator into simple factors, and cancel all factors common to both.

Example.—$\frac{96}{120} = \frac{2\times2\times2\times2\times2\times3}{2\times2\times2\times3\times5} = \frac{2\times2}{5} = \frac{4}{5}$

Here we have cancelled $2\times2\times2\times3$ or 24.

Observe: This $2\times2\times2\times3$ is the G. C. M. of 96 and 120.

Ex. 17. A.—Bring to Lowest Terms, as done above,

$\frac{24}{40}, \frac{16}{30}, \frac{100}{144}, \frac{60}{96}, \frac{56}{84}, \frac{54}{81}, \frac{200}{360}, \frac{98}{490}, \frac{132}{297}.$

2nd Way.—Divide both Numerator and Denominator by any numbers which will exactly divide both of them.

Taking the same fraction $\frac{96}{120}$ we see that we can divide by 3, and then again by 8; thus,

$$3)\ \frac{96}{120} = 8)\ \frac{32}{40} = \frac{4}{5}.$$

3×8 gives us the G. C. M.; as before.

Observe: We may begin by dividing by *any* number which *measures* both 96 and 120.

When no such number can be found the fraction is in its Lowest Terms.

Ex. 17. B.—Bring to Lowest Terms, in the second way,

$\frac{24}{40}, \frac{16}{30}, \frac{100}{144}, \frac{60}{96}, \frac{56}{84}, \frac{54}{81}, \frac{200}{360}, \frac{98}{490}, \frac{132}{297}.$

3rd Way.—Find at once the G. C. M. of the Numerator and Denominator by the Common Rule; then divide by it.

This way is specially adapted to cases of large numbers which cannot easily be resolved into factors, but it may be used in any example.

Example.—$\frac{68}{187}$.

Firstly, find the G. C. M.

```
                    68)187(2
                       136
                       ---
                        51)68(1
                           51
                           --
                           17)51(3
        G. C. M. 17.          51
```

Secondly, cancel thus

$$17)\ \frac{68}{187} = \frac{4}{11}.$$

Ex. 17. C.—Bring to Lowest Terms—

$$\frac{100}{144},\ \frac{98}{490},\ \frac{132}{297},\ \frac{196}{637},\ \frac{68}{1887},\ \frac{341}{3641},\ \frac{123}{410},\ \frac{111}{14874}.$$

An explanation of the fact that when any fraction is brought to Lower Terms its value is not altered.

Take the Fraction $\frac{20}{50}$.

By the rule we can reduce $\frac{20}{50}$ to $\frac{2}{5}$.

We may see that $\frac{2}{5}$ is really the same as $\frac{20}{50}$ in the following way.

Suppose a school of 50 boys:

1 boy is $\frac{1}{50}$, *or one fiftieth,* of the whole school;

20 boys are $\frac{20}{50}$, *or twenty fiftieths,* of the whole school.

Let the school be divided into 5 classes of 10 boys each:

1 class is $\frac{1}{5}$ of the whole school;

2 classes are $\frac{2}{5}$ of the whole school;

That is 20 boys are $\frac{2}{5}$ of the whole school.

As these 20 boys are both $\frac{20}{50}$ and also $\frac{2}{5}$ of the whole school,

It follows that $\frac{20}{50}=\frac{2}{5}$.

Write out this proof from memory; and then

Shew in like manner that $\frac{20}{70}=\frac{2}{7}$.

18. To bring an Improper Fraction to a Mixed Number.

Take as an example the Improper Fraction $\frac{60}{7}$.

$\frac{60}{7}$ *means* "60 divided by 7."

That is "8 and 4 over."

Or as we write it "$8\frac{4}{7}$."

Observe: $8\frac{4}{7}$ is called a mixed number because it is made up of the whole number 8 and the fraction $\frac{4}{7}$.

Ex. 18. A.—Bring to mixed numbers

$$\frac{20}{3},\ \frac{20}{7},\ \frac{20}{11},\ \frac{100}{7},\ \frac{100}{11},\ \frac{200}{33},\ \frac{200}{77},\ \frac{1000}{111}.$$

If, when we divide, there is no remainder the result is a whole number.

$\frac{100}{4}$ means $100\div4$, that is 25, *a whole number.*

Ex. 18. B.—Bring to whole or mixed numbers

$$\frac{200}{4},\ \frac{200}{5},\ \frac{200}{7},\ \frac{200}{9}.$$

It may also happen, in such examples as these, that the fraction can be reduced to Lower Terms;

This should always be done when possible; thus,

Example.—Simplify $\frac{100}{8}$.

$$4)\ \frac{100}{8}=\frac{25}{5}=12\tfrac{1}{2},\ \textit{or}\ \frac{100}{8}=12\tfrac{4}{8}=12\tfrac{1}{2}.$$

Ex. 18. C.—Simplify $\frac{100}{6}$, $\frac{100}{12}$, $\frac{108}{16}$, $\frac{150}{8}$, $\frac{150}{9}$, $\frac{156}{12}$, $\frac{156}{20}$.

It should be noticed that this rule agrees with what we knew before was the case with fractions of a penny.

Thus, 7 farthings, or $\frac{7}{4}$d. $=$ 1 penny 3 farthings, or $1\frac{3}{4}$d.

6 farthings are 1 penny 1 half-penny.

In other words $\frac{6}{4}$d. $= 1\frac{1}{2}$d.

So too it is clear that 9 quarters (of anything) $=$ 2 wholes and 1 quarter; or $\frac{9}{4} = 2\frac{1}{4}$.

Ex. 18. D.—Simplify:—eight halves, ten quarters, eighteen eighths, twenty tenths, sixteen twelfths, eighteen twelfths, thirty twelfths.

19. Mixed Numbers may have to be brought to Improper Fractions,

Just as we may wish to bring $6\frac{3}{4}$d. to farthings.

To bring $6\frac{3}{4}$d. to farthings,

Multiply the 6d. by 4 and add in the 3 farthings.

Ans. 27 farthings *or* $\frac{27}{4}$d.

Example.—Bring $19\frac{3}{5}$ to an improper fraction.

To find the Numerator,

Multiply the 19 by the 5 and add in the 3,

$19 \times 5 + 3 = 98$. *Ans.* $\frac{98}{5}$.

Warning: **The Denominator 5 is *not* changed.**

Ex. 19.—Bring to improper fractions

$11\frac{3}{5}$, $12\frac{4}{7}$, $100\frac{1}{10}$, $1\frac{1}{9}$, $17\frac{3}{14}$.

How many farthings are there in eleven pence three farthings?

How many farthings in eleven pence halfpenny?

How many in eleven pence?

How many *eighths* in *twenty and five eighths ?*

How many in *twenty ?*

Specimen Example.— How many *fifths* are there in *ten and three fifths ?* and how many in *ten ?*

We have $10\frac{3}{5}=\frac{53}{5}$. 1*st Ans.—Fifty-three.*

$10=\frac{50}{5}$. (*See* Step 14.) 2*nd Ans.—Fifty.*

20. A further account of what a Fraction is.

If we speak of $\frac{3}{4}$ of an hour, *or* $\frac{3}{4}$ of a foot, *or* $\frac{3}{4}$ of a penny

It is understood that the whole hour, or foot, or penny is divided into 4 equal parts,

And three of these are taken.

So in every fraction

The Denominator shews into how many equal parts the whole is to be supposed divided;

And the Numerator tells how many of these parts are taken.

1 inch is $\frac{1}{12}$ of a foot.

17 inches are $\frac{17}{12}$ of a foot.

But 17 inches are 1 foot, 5 inches,

That is $1\frac{5}{12}$ feet,

Thus we see how $\frac{17}{12}$ and $1\frac{5}{12}$ are the same thing.

21. To show that the account of the fraction $\frac{3}{4}$ given in (20) is consistent with its meaning

"3 divided by 4."

"$\frac{3}{4}$ of a foot" means "3 times $\frac{1}{4}$ of a foot,"

or "3 × 3 inches," *that is* 9 inches.

But "3 feet divided by 4" = 9 inches.

Thus, "$\frac{3}{4}$ of a foot" and "3 feet divided by 4" are the same thing.

So "$\frac{3}{4}$" and "3 divided by 4" are also the same thing.

QUESTIONS ON STEPS 13—21.

13. What are the two shorter ways of writing down "124 divided by 71"?

14. To what whole numbers are $\frac{14}{1}$, $\frac{14}{2}$, $\frac{14}{7}$, $\frac{14}{14}$, respectively equal?

Express 110 by fractions whose Denominators are respectively 1, 10, 11, 110.

15. Write down two proper fractions, one with 100 as Numerator, the other with 1000 as Denominator.

Also two improper fractions, one with 1000 as Numerator, the other with 100 as Denominator.

16, 17. When may we, as a general rule, cancel, if possible, between two numbers? When is a fraction in its Lowest Terms?

Bring to Lowest Terms $\frac{1008}{2424}$, $\frac{1000}{1550}$, $\frac{3672}{7236}$, $\frac{3610}{19038}$.

Each of these should be proved by doing it in more than one way.

A school of 100 boys is divided into 5 classes of 20 boys each: what fraction of the whole school are 40 boys? and what fraction are 2 classes?

What inference can we draw as to the fractions $\frac{40}{100}$ and $\frac{2}{5}$?

18. Write down four improper fractions, and bring them to mixed numbers.

Write down an improper fraction which can be brought to a whole number, but not to a mixed number.

Simplify *a hundred eighths,* and *thirty quarters.*

19. Bring $19\frac{3}{4}$d. to farthings,
And $4\frac{3}{19}$ to an improper fraction.
How many *ninths* are contained in *four and two ninths ?*
How many in *eight* ?

20. In any fraction
What does the Denominator shew ?
What does the Numerator tell ?

Shew that $\frac{17}{12}$ of a foot $= 1\frac{5}{12}$ ft.
And that $\frac{31}{12}$ of a shilling $= 2\frac{7}{12}$ sh.

21. Shew that "$\frac{3}{4}$ of a foot" is the same as "3 feet divided by 4."

What do we infer as to
"$\frac{3}{4}$" and "3 divided by 4"?

Fractions occurring in answers should now always be given in their lowest terms; Improper Fractions should be brought to mixed numbers.

22. If we wish to add together 3 farthings and 1 halfpenny we first bring the 1 halfpenny to farthings.

We do not say

"3 farthings and 1 halfpenny" or "$\frac{3}{4}$d. + $\frac{1}{2}$d."

But "3 farthings and 2 farthings" or "$\frac{3}{4}$d. + $\frac{2}{4}$d."

We cannot add together $\frac{3}{4}$d. and $\frac{1}{2}$d. without first changing, in our minds, the $\frac{1}{2}$d. into $\frac{2}{4}$d.

3 farthings and 2 farthings make 5 farthings or $1\frac{1}{4}$d.

or $\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$.

Observe: We cannot add together the fractions $\frac{3}{4}$ and $\frac{1}{2}$ *as they stand*, because *they have not the same Denominator.*

As soon as we have changed the second fraction $\frac{1}{2}$ into another equal fraction $\frac{2}{4}$, having the same Denominator, 4, as the first fraction, we can add the two fractions together.

And this we do by *adding the Numerators.*

We write down the Denominator 4 unchanged.

Warning: Never add the Denominators.

If we have to add two fractions which have the same Denominator we simply add the Numerators.

Thus :—add together $\frac{3}{4}$ and $\frac{5}{4}$,

$\frac{3}{4} + \frac{5}{4} = \frac{8}{4} = 2$. *Ans.* 2.

Ex. 22.—(For mental arithmetic if possible.)

Add together $\frac{3}{4}$ and $\frac{1}{2}$; $\frac{1}{4}$ and $\frac{1}{2}$; $\frac{3}{4}$ and $\frac{3}{2}$; $\frac{3}{4}$ and $\frac{9}{4}$; $\frac{3}{2}$ and $\frac{1}{2}$; $\frac{3}{2}$ and $\frac{1}{4}$; $\frac{5}{2}$ and $\frac{1}{4}$.

Specimen Example.—Add $\frac{1}{4}$ and $\frac{3}{2}$.

$\frac{3}{2}$ must be changed into $\frac{6}{4}$,

For we know that 3 halves are the same as 6 quarters,

And our two fractions will then have the same Denominator, 4.

Thus, $\frac{1}{4}+\frac{3}{2}=\frac{1}{4}+\frac{6}{4}=\frac{7}{4}=1\frac{3}{4}$. *Ans.* $1\frac{3}{4}$.

23. To express several different fractions as fractions having the same Denominator,

or, To bring them to a Common Denominator.

Take $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

1*st Step.—To find the Common Denominator.*

Write down the Denominators, and find their L. C. M.

3, 4, 6.

L. C. M. 12.

This 12 is to be the new Denominator of each of the fractions.

2nd Step.—To find the new Numerators.

Take the 12 and divide it by 3, the Denominator of the first fraction; then multiply by the Numerator 2; we thus obtain 8 as the new Numerator of the first fraction, which will now be written $\frac{8}{12}$.

For the second fraction, dividing 12 by 4 and multiplying by 3, we have $\frac{9}{12}$.

And for the third fraction, in the same way, $\frac{10}{12}$.

Ans. $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

Observe: We can shew that $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$ are equal respectively to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ *by bringing them back to their Lowest Terms.*

Ex. 23.—Bring to Common Denominators, $\frac{2}{3}, \frac{1}{4}, \frac{5}{6}$;

$\frac{1}{4}, \frac{3}{5}$; $\frac{3}{4}, \frac{1}{2}, \frac{4}{5}$;

$\frac{11}{12}, \frac{9}{10}, \frac{7}{30}$; $\frac{2}{7}, \frac{8}{9}, \frac{1}{6}, \frac{2}{3}$;

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$; $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{8}, \frac{1}{10}$.

Specimen Example.—$\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{8}, \frac{1}{10}$.

The L.C.M. of 3, 5, 7, 8, 10 is $2 \times 2 \times 2 \times 3 \times 5 \times 7$ *or* 840.

840 will be the Common Denominator: and now to find the new Numerators:—

$$\begin{array}{rrrrr}
3)\underline{840} & 5)\underline{840} & 7)\underline{840} & 8)\underline{840} & 10)\underline{840} \\
280 & 168 & 120 & 105 & 84 \\
\underline{\quad 2} & \underline{\quad 3} & \underline{\quad 4} & \underline{\quad 5} & \underline{\quad 1} \\
560 & 504 & 480 & 525 & 84
\end{array}$$

Ans. $\frac{560}{840}, \frac{504}{840}, \frac{480}{840}, \frac{525}{840}, \frac{84}{840}$.

24. To add together several fractions or mixed numbers.

Example.—Add together $1\frac{1}{2}, 3\frac{2}{3}, \frac{3}{4}, 10\frac{5}{12}, 2$.

1*st Step.*—Take the whole numbers (if there are any) and add them together,

$$1+3+10+2=16.$$

2*nd Step.*—Write down the fractions, leaving out the whole numbers,

Thus, $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{12}$,

and bring them to a Common Denominator by the last rule.

Thus we have, $\frac{6}{12}+\frac{8}{12}+\frac{9}{12}+\frac{5}{12}$,

Since the L. C. M. of 2, 3, 4, 12 is 12.

3rd Step.—Add the Numerators, leaving the Denominator, 12, unchanged, and then reduce;

$$\frac{6+8+9+5}{12}=\tfrac{28}{12}=\tfrac{7}{3}=2\tfrac{1}{3}.$$

Now add in the 16, (which we found at the beginning by adding the whole numbers.)

Thus, $16+2\frac{1}{3}=18\frac{1}{3}$. *Ans.* $18\frac{1}{3}$.

Ex. 24.—Add together $9\frac{1}{6}$, $10\frac{1}{8}$, $\frac{1}{24}$, $\frac{2}{9}$.
$\frac{1}{8}$, $11\frac{1}{3}$, $4\frac{1}{4}$, 3.
$\frac{1}{3}$, $\frac{3}{16}$, $1\frac{11}{48}$, 9, $1\frac{2}{3}$.
$\frac{1}{7}$, $\frac{4}{5}$, $\frac{3}{10}$, $3\frac{1}{14}$, 2.
$\frac{13}{14}$, $\frac{14}{15}$, $\frac{15}{16}$, $\frac{17}{18}$, $\frac{19}{20}$.

25. Subtraction. *(For mental arithmetic.)*

Example.—Subtract $\frac{2}{5}$ from 1.

Think of 1 whole thing divided into 5 equal parts, and 2 of these taken, then 3 remain. *Ans.* $\frac{3}{5}$.

Example.—Subtract $2\frac{3}{7}$ from 3.

Think of 2 weeks and 3 days, taken from 3 weeks; for 3 days $=\frac{3}{7}$ of a week; we have left 4 days or $\frac{4}{7}$ of a week.

Ans. $\frac{4}{7}$.

Example.—Subtract $6\frac{3}{8}$ from $10\frac{1}{4}$.

Think of 6 pence and 3 mites (a mite being $\frac{1}{8}$ of a penny,) subtracted from 10 pence and 1 farthing. *Ans.* $3\frac{5}{8}$.

Ex. 25.—Subtract $\frac{3}{7}$ from 1; $\frac{3}{8}$ from 2; $\frac{3}{10}$ from 3.

Simplify $7-3\frac{1}{3}$; $8\frac{1}{2}-3\frac{1}{4}$; $8\frac{1}{4}-6\frac{1}{2}$;
$11-7\frac{2}{11}$; $7\frac{2}{11}-5$; $3\frac{1}{2}-\frac{3}{8}$.

Warning: A−B does *not* mean "A *subtracted from* B" but "B *subtracted from* A."

26. To subtract one fraction or mixed number from another.

Example.—Subtract $4\frac{1}{5}$ from $5\frac{1}{4}$.

Write down $5\frac{1}{4}-4\frac{1}{5}$, *not* $4\frac{1}{5}-5\frac{1}{4}$.

Bring the mixed numbers to improper fractions; thus,

$$5\tfrac{1}{4}-4\tfrac{1}{5}=\tfrac{21}{4}-\tfrac{21}{5}.$$

$$=\frac{105-84}{20}=\tfrac{21}{20}.$$

$$=1\tfrac{1}{20}.\quad \textit{Ans.}\ 1\tfrac{1}{20}.$$

Observe: Subtraction is like Addition, except that

(i) We bring the mixed numbers to improper fractions before we begin; and

(ii) We subtract the second Numerator instead of adding it.

Ex. 26.—Subtract $\frac{3}{10}$ from $1\frac{1}{2}$; $1\frac{1}{2}$ from $2\frac{1}{9}$; $2\frac{1}{9}$ from $7\frac{1}{6}$.

Simplify $1\frac{3}{5}-\frac{7}{10}$; $11\frac{10}{11}-10\frac{13}{22}$; $19\frac{1}{2}-3\frac{15}{16}$; $8\frac{2}{3}-2\frac{3}{8}$; $11\frac{1}{9}-10\frac{1}{6}$; $4\frac{5}{12}-3\frac{7}{10}$; $100\frac{1}{11}-11\frac{1}{100}$.

27. To simplify a row of fractions connected by + or −, such as

$$1\tfrac{1}{2}-3\tfrac{1}{3}+4\tfrac{3}{4}-2+\tfrac{1}{6}.$$

We bring all the mixed numbers to improper fractions,

Writing the whole number 2 as $\frac{2}{1}$.

Thus we have, $\frac{3}{2}-\frac{10}{3}+\frac{19}{4}-\frac{2}{1}+\frac{1}{6}$.

The L. C. M. of 2, 3, 4, 1, 6, is 12.

12 then will be the Common Denominator.

We find the new Numerators as before, and write them thus,

$$\frac{18-40+57-24+2}{12}$$

Keeping the + and − signs in their proper places.

Now $18-40+57-24+2$
$=18+57+2-40-24$
$=77-64=13.$

We have then $\frac{13}{12}=1\frac{1}{12}$. *Ans.* $1\frac{1}{12}$.

Warning: $2-5+4$ is 1, being the same as $2+4-5$.

Ex. 27.—(i) To what are each of the following equal:—

$$3-6+5;\ 6-5+4-2;\ 1-2+3?$$

(ii) Simplify each of the following:—

$$1\tfrac{1}{2}+3\tfrac{1}{3}-4\tfrac{3}{4}+2-\tfrac{1}{6};\ 1\tfrac{1}{2}+3\tfrac{1}{3}+4\tfrac{3}{4}-2-\tfrac{1}{6}.$$

Remember how whole numbers are to be expressed.

(iii) Simplify $3\frac{1}{6}-2\frac{1}{12}+1\frac{1}{4}$; $12\frac{1}{8}-13\frac{1}{3}+4\frac{1}{12}$;
$3-8\frac{2}{7}+5\frac{1}{9}+\frac{40}{63}$; $\frac{7}{16}-1\frac{5}{16}+2\frac{5}{8}$;
$4\frac{2}{9}+4\frac{1}{12}-7-\frac{5}{6}$; $4\frac{4}{9}-5\frac{5}{9}+1\frac{5}{18}$.

Answers to (iii) $2\frac{1}{3}$; $2\frac{7}{8}$; $\frac{29}{63}$; $1\frac{3}{4}$; $\frac{17}{36}$; $\frac{1}{6}$.

28. Multiplication.—Half of a half is a quarter,

or $\frac{1}{2}$ of $\frac{1}{2}=\frac{1}{4}$.

It must be understood that "of" is equivalent to "multiplied by," which is expressed by "×."

A score *of* sixpences = 6d. *multiplied by* 20.

Thus $\frac{1}{2}\times\frac{1}{2}$ is the same as $\frac{1}{2}$ of $\frac{1}{2}$,
and so is equal to $\frac{1}{4}$.

Again $\frac{3}{4}$ of a shilling = 9d.

So $\frac{2}{3}$ of ($\frac{3}{4}$ of a shilling) = $\frac{2}{3}$ of 9d. = 6d.

And 6d. = $\frac{6}{12}$ of a shilling.

$\therefore$ $\frac{2}{3}$ of $\frac{3}{4}$ of a shilling = $\frac{6}{12}$ of a shilling,

or $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}$.

When then we multiply together, as here, two or more fractions, we multiply together the Numerators for a new Numerator, and the Denominators for a new Denominator.

Mixed numbers must be first brought to Improper Fractions.

Ex. 28.—Find the values of

$$\tfrac{1}{2} \text{ of } \tfrac{1}{3}\,;\ \ \tfrac{1}{2} \text{ of } \tfrac{3}{4}\,;\ \ \tfrac{1}{3} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{11}{13}\,;$$

$$\tfrac{8}{9} \times 2\tfrac{1}{3} \times 1\tfrac{3}{5}\,;\ \ \tfrac{7}{9} \text{ of } 1\tfrac{2}{5} \times \tfrac{2}{5}\,;\ \ 1\tfrac{1}{2} \times 1\tfrac{1}{4} \times 1\tfrac{1}{8}.$$

Remember that "of" means the same as "×."

Specimen Example.—

$$\tfrac{8}{9} \text{ of } 1\tfrac{2}{5} \times 2\tfrac{2}{3} = \tfrac{8}{9} \text{ of } \tfrac{7}{5} \times \tfrac{8}{3} = \frac{8 \times 7 \times 8}{9 \times 5 \times 3}$$

$$= \tfrac{448}{135} = 3\tfrac{43}{135}. \quad \textit{Ans. } 3\tfrac{43}{135}.$$

29. An example like the following

$$\tfrac{3}{4} \times \tfrac{4}{5} \times \tfrac{5}{9} = \frac{3 \times 4 \times 5}{4 \times 5 \times 9} = {}^{10})\tfrac{60}{180} = {}^{6})\tfrac{6}{18} = \tfrac{1}{3}$$

can be done more shortly, thus,

$$\tfrac{3}{4} \times \tfrac{4}{5} \times \tfrac{5}{9} = \tfrac{1}{1} \times \tfrac{1}{1} \times \tfrac{1}{3} = \tfrac{1}{3}.$$

The two 4's are *cancelled* against one another;
so are the two 5's;
and the 3 and the 9 are each divided by 3.

Example.—Simplify $3\tfrac{1}{3}$ of $\tfrac{2}{5}$ of $\tfrac{3}{4}$.

$3\tfrac{1}{3}$ of $\tfrac{2}{5}$ of $\tfrac{3}{4}$
$= \tfrac{10}{3}$ of $\tfrac{2}{5}$ of $\tfrac{3}{4}$
$= \tfrac{2}{1}$ of $\tfrac{2}{1}$ of $\tfrac{1}{4}$ cancelling between 3 and 3 and between 10 and 5.
$= \tfrac{1}{1}$ of $\tfrac{1}{1}$ of $\tfrac{1}{1}$ cancelling between 2 × 2 and 4.
$= 1.$

Remember that when, as here, all the numbers cancel out, the result is not 0 but 1.

Ex. 29.—Simplify $\frac{3}{4}$ of $\frac{8}{11}$; $\frac{3}{4}$ of $1\frac{1}{7}$; $1\frac{1}{2}$ of $1\frac{1}{6}$; $1\frac{1}{2}$ of $\frac{1}{6}$ of 12;
$1\frac{3}{4}$ of $\frac{2}{7}\times 2\times\frac{2}{3}$ of $1\frac{1}{2}$;
$1\frac{1}{2}$ of $1\frac{1}{3}$ of $1\frac{1}{4}$ of $1\frac{1}{5}\times 3$;
$1\frac{1}{7}$ of $1\frac{3}{4}\times 7$; $3\frac{3}{4}$ of 8 of $1\frac{3}{5}$; 12 of $1\frac{1}{8}\times\frac{1}{12}$ of $\frac{8}{9}$.

Warning: Never cancel between two numbers which are to be multiplied together, such as two Numerators or two Denominators.

Write whole numbers, if they occur, as fractions whose Denominator is 1.

Specimen Example.—$1\frac{3}{4}\times 7\times\frac{4}{35}\times 5$,
$=\frac{7}{4}\times\frac{7}{1}\times\frac{4}{35}\times\frac{5}{1}$,
$=\frac{7}{1}$. *Ans.* 7.

30. Division.—"12 divided by 3" means
"The number of times 3 is contained in 12."

So "$1\frac{1}{2}\div\frac{1}{2}$" means
"The number of times $\frac{1}{2}$ is contained in $1\frac{1}{2}$."

This we know is 3.

That is $1\frac{1}{2}\div\frac{1}{2}=3$.

This result 3 is found by the following rule:—

Turn the Divisor, *i. e.* $\frac{1}{2}$, upside down, *i. e.* make it $\frac{2}{1}$, then multiply, thus,

$$1\frac{1}{2}\div\frac{1}{2}=1\frac{1}{2}\times\frac{2}{1}=\frac{3}{2}\times\frac{2}{1}=\frac{3}{1}=3.$$

Remember that mixed numbers must be expressed at once as improper fractions, and whole numbers, as 7, must be expressed in this way, $\frac{7}{1}$.

Ex. 30.—Divide $3\frac{1}{2}$ by 7; $4\frac{1}{2}$ by $\frac{1}{2}$; $5\frac{1}{3}$ by $1\frac{1}{3}$.

Simplify $11\frac{1}{2}\div 5\frac{3}{4}$; $5\frac{3}{4}$ by $11\frac{1}{2}$; $10\frac{1}{9}\div 9\frac{1}{10}$; $8\frac{1}{7}\div 7\frac{1}{8}$.

Specimen Ex.—$10\frac{1}{9}\div 9\frac{1}{10}=\frac{91}{9}\div\frac{91}{10}=\frac{91}{9}\times\frac{10}{91}=\frac{10}{9}=1\frac{1}{9}$.

Ans. $1\frac{1}{9}$.

Warning: Never forget to change ÷ into × when you turn the divisor upside down.

QUESTIONS ON STEPS 22—30.

22. Why cannot the fractions $\frac{1}{2}$ and $\frac{1}{4}$ be added *as they stand?*

23. Write down the rule for bringing fractions to a Common Denominator.

Bring to a Common Denominator 7, $7\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{14}$.

24—27. What difference should be observed between the methods of adding and subtracting Mixed Numbers?

Simplify $5-6+3$; $2\frac{1}{5}+3\frac{1}{6}+4\frac{1}{15}$; $2\frac{1}{5}-3\frac{1}{6}+4\frac{1}{15}$.

Write down, without working out on paper, the results of $\frac{1}{4}+\frac{3}{8}$; $\frac{3}{8}-\frac{1}{4}$; $1\frac{1}{2}+\frac{5}{8}$; $1\frac{1}{2}-\frac{5}{8}$; $9+\frac{1}{3}+\frac{1}{6}$; $9-\frac{1}{3}-\frac{1}{6}$.

28. To what is the word "of" to be considered as equivalent?

Shew that $\frac{2}{3}$ of $\frac{3}{4}$ of a shilling $=\frac{6}{12}$ of a shilling.

29. What directions are to observed in cancelling? and what mistakes are to be avoided?

30. Give the rule for dividing one fraction by another.

How should whole and mixed numbers respectively be expressed?

Simplify $2\frac{2}{3}$ of $3\frac{2}{4}\div 7\frac{7}{8}$.

31. Complex Fractions.—

We know that $20 \div 5$ may be expressed as $\frac{20}{5}$.

So too $\frac{1}{3} \div \frac{1}{2}$ may be expressed as $\dfrac{\frac{1}{3}}{\frac{1}{2}}$.

$\dfrac{\frac{1}{3}}{\frac{1}{2}}$ is what is called a Complex Fraction.

Simplify the Complex Fraction $\dfrac{3\frac{1}{3}}{1\frac{1}{9}}$

$$\frac{3\frac{1}{3}}{1\frac{1}{9}} \text{ means } 3\tfrac{1}{3} \div 1\tfrac{1}{9}$$

$$= \tfrac{10}{3} \div \tfrac{10}{9} = \tfrac{10}{3} \times \tfrac{9}{10} = \tfrac{3}{1}. \quad \textit{Ans. } 3.$$

Remember that we do not give $\frac{3}{1}$ as our answer, but 3.

Ex. 31.—Simplify $\dfrac{\frac{4}{5}}{1\frac{1}{2}}, \dfrac{3\frac{1}{2}}{1\frac{1}{4}}, \dfrac{11}{11\frac{1}{2}}, \dfrac{13\frac{1}{2}}{2}, \dfrac{3\frac{1}{9}}{9\frac{1}{3}}, \dfrac{13}{5\frac{1}{5}}$.

Remember that, in the 3rd Example, the Numerator 11 must be written $\frac{11}{1}$; and, in the 4th, the Denominator 2 must be written $\frac{2}{1}$, &c.

Specimen Example.—$\dfrac{13}{5\frac{1}{5}} = \dfrac{\frac{13}{1}}{\frac{26}{5}} = \frac{13}{1} \div \frac{26}{5} = \frac{13}{1} \times \frac{5}{26}$

$$= \tfrac{5}{2} = 2\tfrac{1}{2}. \quad \textit{Ans. } 2\tfrac{1}{2}.$$

32. When a bracket occurs, all that is inside the bracket must be simplified *first*.

Simplify $1\frac{1}{2} \div (\frac{1}{3} \times 1\frac{1}{5}.)$

Take first what is within the bracket,

$$\tfrac{1}{3} \times 1\tfrac{1}{5} = \tfrac{1}{3} \times \tfrac{6}{5} = \tfrac{2}{5}.$$

Now writing $\frac{2}{5}$ for $(\frac{1}{3} \times 1\frac{1}{5})$ in the original quantity,

$$1\tfrac{1}{2} \div \tfrac{2}{5} = \tfrac{3}{2} \times \tfrac{5}{2} = \tfrac{15}{4} = 3\tfrac{3}{4}. \qquad \textit{Ans. } 3\tfrac{3}{4}.$$

When "of" stands between two or more fractions they are to be considered *as in a bracket,* and multiplied together before the rest is done.

Thus, $\frac{1}{2}$ of $\frac{1}{3} \div \frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5} = (\frac{1}{2} \times \frac{1}{3}) \div (\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5})$

$= \frac{1}{6} \div \frac{1}{60} = \frac{1}{6} \times \frac{60}{1} = 10.$

Ex. 32.—Simplify $\frac{1}{2} \div (\frac{1}{3} \times 1\frac{1}{2})$; $2 \times (1\frac{1}{2} \div \frac{3}{4})$;

$(3\frac{1}{2} \div \frac{1}{4}) \times (2\frac{1}{3} \div 3)$; $3\frac{1}{2} \div (\frac{1}{4} \times 2\frac{1}{3} \div 3)$;

$3\frac{1}{2} \div \frac{1}{4}$ of $1\frac{1}{3}$; $3\frac{1}{2}$ of $\frac{1}{4} \div 1\frac{1}{3}$;

$\frac{1}{3}$ of $\frac{1}{4} \div (\frac{1}{2} \div \frac{1}{3})$; $\frac{1}{3}$ of $(\frac{1}{4} \div \frac{1}{2}) \div \frac{1}{3}$;

$2 \div \frac{3}{4}$ of $\frac{1}{3}$ of $1\frac{1}{3}$; $\frac{1}{2}$ of $4 \times (1\frac{1}{3} \div \frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4})$;

$\frac{1}{3}$ of $6 \times (\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4} \div \frac{2}{3})$.

Specimen Example.—$\frac{1}{3}$ of $4 \times (\frac{2}{3} \div \frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4})$

$= \frac{4}{3} \times (\frac{2}{3} \div \frac{1}{24})$

$= \frac{4}{3} \times (\frac{2}{3} \times \frac{24}{1}) = \frac{4}{3} \times \frac{16}{1} = \frac{64}{3} = 21\frac{1}{3}.$

Ans. $21\frac{1}{3}$.

Answers to the last five Examples, $\frac{1}{18}$, $\frac{1}{2}$, 6, 64, $\frac{1}{8}$.

Ex. 33.—Simplify $\dfrac{1\frac{1}{2}+2\frac{2}{3}-3\frac{1}{4}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}$, $\dfrac{2-4\frac{1}{3}+3\frac{3}{4}}{8-\frac{7}{9}+\frac{5}{6}}$, $\dfrac{\frac{3}{4}-\frac{4}{5}+\frac{5}{6}}{\frac{5}{6}+\frac{4}{5}-\frac{3}{4}}$.

Warning: 3 − 4 + 5 is *not* the same as 3 − 9.

Specimen Example.— $\dfrac{2-4\frac{1}{3}+3\frac{3}{4}}{\frac{7}{9}-\frac{5}{6}+8}$

$$= \frac{\frac{2}{1}-\frac{13}{3}+\frac{15}{4}}{\frac{7}{9}-\frac{5}{6}+\frac{8}{1}}$$

$$= \frac{\dfrac{24-52+45}{12}}{\dfrac{14-15+144}{18}}$$

$$=\frac{\frac{17}{12}}{\frac{143}{18}}$$

$$=\frac{17}{12}\div\frac{143}{18}=\frac{17}{12}\times\frac{18}{14}=\frac{51}{286}.\ \textit{Ans.}\ \frac{51}{286}.$$

Answers to Ex. 33, $2\frac{1}{5}$, $\frac{51}{290}$, $\frac{47}{53}$.

Always arrange the working in this way : placing the =’s underneath one another, as long as we have a complex fraction.

Ex. 34.—Simplify $\frac{1\frac{1}{2}\text{ of }3\frac{1}{3}-4\frac{1}{4}}{1\frac{1}{4}\text{ of }(4\frac{1}{4}-3\frac{1}{3})}$; $\frac{\frac{3}{4}\div(2\frac{1}{2}-1\frac{1}{3})}{(\frac{3}{4}\div2\frac{1}{2})+1\frac{1}{3}}$; $\frac{5-1\frac{5}{11}}{5\div1\frac{5}{11}}$; $\frac{11\frac{1}{3}\div8\frac{1}{2}}{3\frac{1}{11}\div2\frac{1}{8}}$; $\frac{1\frac{3}{4}\text{ of }\frac{2}{7}-\frac{1}{3}+3\frac{1}{8}}{(1\frac{3}{4}\div\frac{2}{7})\div4\frac{9}{10}}$; $\frac{\frac{2}{3}\times\frac{3}{4}\div\frac{4}{5}}{\frac{2}{3}+\frac{3}{4}-\frac{4}{5}}$; $\frac{\frac{1}{4}\text{ of }\frac{1}{3}+(\frac{1}{4}\div\frac{1}{3})}{(\frac{1}{4}+\frac{1}{3})\div(\frac{1}{3}-\frac{1}{4})}$.

Specimen Example.— $$\frac{2\frac{1}{2}\text{ of }3\frac{1}{4}-4\frac{1}{6}}{2\frac{1}{2}\div(4\frac{1}{6}-3\frac{1}{4})}$$

$$=\frac{\frac{5}{2}\text{ of }\frac{13}{4}-\frac{25}{6}}{\frac{5}{2}\div(\frac{25}{6}-\frac{13}{4})}$$

$$=\frac{\frac{65}{8}-\frac{25}{6}}{\frac{5}{2}\div\frac{50-39}{12}}$$

$$=\frac{\frac{195-100}{24}}{\frac{5}{2}\div\frac{11}{12}}$$

$$=\frac{\frac{95}{24}}{\frac{30}{11}}$$

$$=\frac{95}{24}\times\frac{11}{30}=\frac{209}{144}.\quad\textit{Ans.}\ 1\frac{65}{144}.$$

Answers to Ex. 34, $\frac{6}{11}$; $\frac{135}{343}$; $\frac{11}{12}$; $2\frac{19}{30}$; $1\frac{1}{74}$; $\frac{5}{42}$; $\frac{39}{80}$.

Ex. 35.—Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$ of $\dfrac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{2}\div\frac{1}{3}}$; $\dfrac{\frac{1}{3}\div\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}} \div \dfrac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}$;

$\dfrac{8\frac{1}{2}\div 34}{34-8\frac{1}{2}}$ of $3\frac{3}{4}$; $\dfrac{\frac{1}{2}-\frac{1}{5}}{\frac{1}{2}\times\frac{1}{5}}$ of $\dfrac{\frac{1}{2}\div\frac{1}{5}}{\frac{1}{2}+\frac{1}{5}}$; $\dfrac{8\div\frac{7}{8}}{7\frac{1}{9}}$ of $\dfrac{7\div\frac{6}{7}}{6\frac{1}{8}}$ of $\dfrac{6\div\frac{5}{6}}{5\frac{1}{7}}$.

Specimen Example.—$\dfrac{\frac{1}{3}-\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}} \div \dfrac{\frac{1}{3}\div\frac{1}{5}}{\frac{1}{3}+\frac{1}{5}}$.

Simplifying the first Complex Fraction, we have

$$\frac{\frac{1}{3}-\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}}=\frac{\frac{5-3}{15}}{\frac{1}{15}}=\frac{2}{15}\div\frac{1}{15}=\frac{2}{1}.$$

Simplifying the second, we have

$$\frac{\frac{1}{3}\div\frac{1}{5}}{\frac{1}{3}+\frac{1}{5}}=\frac{\frac{1}{3}\times\frac{5}{1}}{\frac{5+3}{15}}=\frac{5}{3}\div\frac{8}{15}=\frac{5}{3}\times\frac{15}{8}=\frac{25}{8}.$$

Dividing the first result by the second result, we have,

$\frac{2}{1}\div\frac{25}{8}=\frac{2}{1}\times\frac{8}{25}=\frac{16}{25}$. *Ans.* $\frac{16}{25}$.

Answers to Ex. 35, $\frac{5}{9}$; 112; $\frac{10}{17}$; $10\frac{5}{7}$; $2\frac{2}{5}$.

Warning : Never cancel *into part only* of a Numerator or Denominator, *that is*, NEVER CANCEL AS BELOW,

$$\frac{3+\frac{1}{8}}{3-\frac{1}{7}}=\frac{1+\frac{1}{8}}{1-\frac{1}{7}};\qquad \frac{3\frac{1}{8}}{3\frac{1}{2}+3\frac{1}{4}}=\frac{1\frac{1}{8}}{1\frac{1}{2}+1\frac{3}{4}}.$$

We may however cancel thus,

$$\frac{8\times\frac{1}{3}}{8\div\frac{1}{3}}=\frac{1\times\frac{1}{3}}{1\div\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{3}{1}}=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}.$$

For here 8 is a factor of both the Numerator and the Denominator.

Or thus, $\dfrac{\frac{3}{4}\text{ of }\frac{4}{5}}{\frac{2}{5}}=\dfrac{\frac{3}{1}}{\frac{2}{1}}=1\frac{1}{2}$.

36. Add together £2 0s. $1\frac{7}{8}$d., £1 1s. $11\frac{7}{9}$d., $6\frac{1}{2}$d.

£.	s.	d.
2	0	$1\frac{7}{8}$
1	1	$11\frac{7}{9}$
0	0	$6\frac{1}{2}$
3	2	$8\frac{11}{12}$

We add the fractions thus,

$$\frac{7}{8}+\frac{7}{9}+\frac{1}{2}$$
$$=\frac{63+56+36}{72}=\frac{155}{72}=2\frac{11}{72}.$$

So put down $\frac{11}{72}$ and carry 2.

Ans. £3 2s. $8\frac{11}{72}$d.

Subtract 1£1 9s. $11\frac{7}{9}$d. from £11 8s. $0\frac{1}{3}$d.

£.	s.	d.
11	8	$0\frac{1}{3}$
1	19	$11\frac{7}{9}$
9	8	$0\frac{5}{9}$

We cannot take $\frac{7}{9}$ from $\frac{1}{3}$ since $\frac{7}{9}$ is greater than $\frac{1}{3}$, so we borrow 1, and say

$$1\frac{1}{3}-\frac{7}{9}=\frac{4}{3}-\frac{7}{9}=\frac{12-7}{9}=\frac{5}{9},$$

We put down $\frac{5}{9}$ and remember that we have borrowed 1.

Ans. £9 8s. $0\frac{5}{9}$d.

Ex. 36. A.—Add together

i. £8 8s. $10\frac{7}{1}$d.; 8s. $0\frac{8}{13}$d.; £5 0s. $3\frac{9}{10}$d.

ii. 5 days, $1\frac{1}{9}$ hr.; 6 days, $2\frac{2}{7}$ hrs.; 11 days, $0\frac{20}{21}$ hr.

iii. 1 ft. $11\frac{3}{4}$ in.; 4 ft. $11\frac{2}{3}$ in.; 2 ft. $0\frac{11}{12}$ in.

iv. 8 acres, $1\frac{1}{9}$ rood; 1 a. $0\frac{9}{10}$ r.; 3 acres, $2\frac{71}{90}$ roods.

Ex. 36. B.—Subtract £8 0s. $0\frac{7}{8}$d. from £9 8s. $2\frac{1}{2}$d.
6 weeks, 6 days, $16\frac{11}{56}$ hours from 8 weeks 2 days.
8 yds. 0ft. $11\frac{11}{12}$ in. from 20 yds. 1 ft. $0\frac{1}{3}$ in.

The adding or subtracting of the fractions should always be shewn.

37. Multiply £21 1s. $4\frac{2}{17}$d. by 11.

$$\begin{array}{rrr} 21 & 1 & 4\frac{2}{7} \\ & & 11 \\ \hline 231 & 14 & 11\frac{1}{7} \end{array}$$

Ans. £231 14s. $11\frac{1}{7}$d.

Here 11 times $\frac{2}{7}$ is $\frac{22}{7}$, or $3\frac{1}{7}$,
Put down $\frac{1}{7}$ and carry 3.

To multiply by a large number, 103 for instance, follow the usual method; multiply by 10 and 10, and 3 times the top line; and remember in each case to multiply the fraction as is done above.

Divide £8 7s. $8\frac{4}{9}$d. by 7.

$$\begin{array}{l} 7\underline{)8 \quad 7 \quad 8\frac{4}{9}\qquad} \\ \quad 1 \quad 3 \quad 11 \textit{ and } 3\frac{4}{9} \textit{ over.} \end{array}$$

Do not bring the 3d. *to farthings,*
But say "$3\frac{4}{9} \div 7 = \frac{34}{9} \times \frac{1}{7} = \frac{34}{63}$."

Ans. £1 3s. $11\frac{34}{63}$d.

Ex. 37. A.—Multiply
£11 0s. $4\frac{1}{7}$d. by 11; £123 12s. $0\frac{5}{16}$d. by 10;
£9 1s. $2\frac{2}{9}$d. by 8; £3 0s. $0\frac{11}{90}$d. by 9.

Divide
£111 0s. $4\frac{1}{7}$d. by 7, 11, 8, 5, and 9.

Divide 18s. $4\frac{2}{17}$d. by 13.

$$\begin{array}{r l} & \text{s.}\quad \text{d.} \\ 13) & 18 \quad 4\frac{2}{17}(1 \\ & \underline{13} \\ & \;5 \\ & \underline{12} \\ 13) & 64(4 \\ & \underline{52} \\ & 12 \end{array}$$

Do not bring this remainder, 12, *to farthings,*
But go on thus,

$$12\frac{2}{17} \div 13 = \frac{206}{17} \times \frac{1}{13} = \frac{206}{221}.$$

Ans. £1 12s. $4\frac{206}{221}$d.

Ex. 37. B.—Multiply £601 5s. $2\frac{2}{5}$d. by 81 and by 18; also divide it by 91 and by 19.
Multiply 409 yds. 0 ft. $11\frac{1}{13}$ in. by 71 and by 37; also divide it by 81 and by 43.
Multiply 3 weeks, 4 days, $2\frac{10}{11}$ hours by 19 and by 91; also divide it by 77 and by 107.

38. "12 multiplied by $\frac{3}{4}$"
means "12 multiplied by 3 divided by 4."
We may multiply the 12 by the 3 *first,*
and then divide by the 4.

Thus,
$$\begin{array}{r r} & 12 \\ & \underline{3} \\ 4) & 36 \\ & 9 \end{array} \qquad \textit{Ans. } 9.$$

It should be noticed that this is the same result as we get by doing the example this way

$$12 \times \tfrac{3}{4} = \tfrac{12}{1} \times \tfrac{3}{4} = \tfrac{9}{1} = 9.$$

When then we wish to multiply by a fraction we may
multiply by the Numerator of the fraction,
and then divide by the Denominator.

Example.—Multiply £3 6s. $9\frac{3}{4}$d. by $\frac{2}{3}$.

$$\begin{array}{r|rrr} & 3 & 6 & 9\frac{3}{4} \\ & & & 2 \\ \hline 3) & 6 & 13 & 7\frac{1}{2} \\ \hline & 2 & 4 & 6\frac{1}{2} \end{array}$$

Ans. £2 4s. $6\frac{1}{2}$d.

Remembering that "of" means "multiplied by,"
we understand that such an example as

Find $\frac{3}{5}$ of £1 6s. $4\frac{1}{4}$d.

is done *in exactly the same way as the last.*

If we have such an example as

£11 0s. $2\frac{3}{4}$d. $\div \frac{11}{12}$

we change it into £11 0s. $2\frac{3}{4}$d. $\times \frac{12}{11}$.

Remember to change ÷ into × when you write the fraction upside down.

Obs.—If we have to divide *(for example,)* £8 4s. 2d. by $2\frac{2}{3}$.
we *must* first bring the $2\frac{2}{3}$ to the improper fraction $\frac{8}{3}$,
we have then £8 4s. 2d. $\times \frac{3}{8}$,
and this can be done by the rule given above.

Example.—Find the value of $3\frac{5}{8}$ of £1 2s. 8d.

As this is not an example in division we are *not* obliged to bring $3\frac{5}{8}$ to an improper fraction, but may do it thus,

$$\begin{array}{rrr} \text{£.} & \text{s.} & \text{d.} \\ 1 & 2 & 8 \\ & & 3 \\ \hline 3 & 8 & 0 \end{array} \qquad\qquad \begin{array}{r|rrr} & \text{£.} & \text{s.} & \text{d.} \\ & 1 & 2 & 8 \\ & & & 5 \\ \hline 8) & 5 & 13 & 4 \\ \hline & & 14 & 2 \end{array}$$

Here we have multiplied by the 3. Here by the $\frac{5}{8}$.

Now we add the two results together,

£.	s.	d.
3	8	0
0	14	2
4	2	2

Ans. £4 2s. 2d.

Ex. 38.—Multiply £7 8s. $6\frac{2}{7}$d. by 8, by $9\frac{1}{3}$, and by $10\frac{3}{5}$.

Divide £7 8s. $6\frac{3}{7}$d. by 11, by $12\frac{1}{2}$, and by $13\frac{1}{3}$.

Find the value of $8\frac{3}{14}$ of 2 weeks 1 day, and of $19\frac{11}{12}$ of 1 ton, 8 cwts.

Multiply 7 tons 7 cwts. 0 qrs. $7\frac{7}{17}$lbs. by $\frac{70}{71}$ and by $1\frac{1}{70}$, and prove each result by division.

39. What fraction is 3 of 4?

There is no difficulty in seeing that

$$3 = \frac{3}{4} \text{ of } 4.$$

In other words, if the question is asked

"What fraction is 3 of 4"?

the answer is "$\frac{3}{4}$."

But if the question is asked

"What is 3 divided by 4"?

the only answer that we can give is this same fraction "$\frac{3}{4}$."

We must understand then that

(supposing A and B to stand for numbers)

The question "*What fraction is* A *of* B"? means the same as "*What is* A *divided by* B"? and the answer is the raction $\frac{A}{B}$.

Example.—Express £8 0s. 1d. as the fraction of £12 6s.

The rule simply tells us to write down as a fraction

$$\frac{\pounds 8\ \ 0\text{s.}\ \ 1\text{d.}}{\pounds 12\ \ 6\text{s.}\ \ 0\text{d.}}$$

But as we have here not abstract numbers but £. s. d. we first bring to a common Denomination

£8 0s. 1d. = 1921 pence ; £12 6s. = 2952 pence ;

The answer then is $\frac{1921}{2952}$.

Example.—Express $1\frac{3}{4}$ of 3s. 6d. as the fraction of $3\frac{1}{4}$ of 2s. 1d.

Bring 3s. 6d. and 2s. 1d. to pence.

Thus we have " $1\frac{3}{4}$ of 42d. as the fraction of $3\frac{1}{4}$ of 25d."

Here then, by the rule, we write $1\frac{3}{4}$ of 42 as the Numerator of a fraction, and $3\frac{1}{4}$ of 25 as the Denominator, and then simplify—

$$\frac{1\frac{3}{4} \text{ of } 42}{3\frac{1}{4} \text{ of } 25} = \frac{\frac{7}{4} \text{ of } \frac{42}{1}}{\frac{13}{4} \text{ of } \frac{25}{1}} = \frac{294}{325}. \quad \textit{Ans. } \frac{294}{325}.$$

Ex. 39.—(i) Express 3s. as the fraction of 4s. ;

4s., 4s. 6d., 4s. 9d. each as the fraction of 5s. ;

£1 7s. 6d., £1 19s. 0d., £1 0s. $0\frac{1}{2}$d. each as the fraction of £3.

Warning : In such an example as

" Express £1 17s. 6d. as the fraction of £2,"

Observe : It is *absolutely necessary* to bring the two sums of money to the *same* denomination.

And it is best that this denomination should be as large as possible, thus we should not bring £1 17s. 6d. and £2 to pence, but to half-crowns.

(ii) Express $\frac{1}{3}$ of $3\frac{3}{4}$ of a day as the fraction of 12 hrs. and $\frac{1}{4}$ of $\frac{2}{9}$ of a week as the fraction of $\frac{1}{12}$ of 28 days.

(iii) What fraction of $\frac{1}{3}$ of $3\frac{1}{2}$ yds. is 1 ft. $7\frac{1}{12}$ in. ?
What fraction of $\frac{6}{7}$ of 3 guineas is £1 1s. $1\frac{1}{4}$d. ?
What fraction of 3 guineas is $\frac{6}{7}$ of £1 1s. $1\frac{1}{4}$d. ?
What fraction of 1 inch is $\frac{1}{1000000}$ of 1 mile ?

Specimen Example.—What fraction of $\frac{1}{4}$ of $4\frac{4}{5}$ of a day is $\frac{1}{8}$ of 12 hours?

This means

Express $\frac{1}{8}$ of 12 hours as the fraction of $\frac{1}{4}$ of $4\frac{4}{5}$ days.

$$\frac{\frac{1}{8}\text{ of 12 hours}}{\frac{1}{4}\text{ of }4\frac{4}{5}\text{ days}}=\frac{\frac{1}{2}\text{ of 12 hours}}{\frac{1}{4}\text{ of }4\frac{4}{5}\text{ of 24 hrs.}}$$

$$=\frac{\frac{1}{2}\text{ of }\frac{12}{1}}{\frac{1}{4}\text{ of }\frac{24}{5}\text{ of }\frac{24}{1}}=\frac{\frac{6}{1}}{\frac{4}{5}}=\frac{5}{24}.\quad \textit{Ans. } \frac{5}{24}.$$

Warning : Do not say "$\frac{5}{24}$ of an hour;" read over the question and you will see that it would be nonsense.

QUESTIONS ON STEPS 31—39.

31. Express as a Complex Fraction $8\frac{1}{2} \div 2\frac{1}{8}$.

And give, in another form, the fraction $\dfrac{1\frac{1}{2}}{2\frac{1}{3}}$.

32. What directions are to be observed when brackets occur?

Simplify $\frac{1}{2} \div \frac{1}{3}$ of $(\frac{1}{4} \div \frac{1}{5})$; $(\frac{1}{2} \div \frac{1}{3})$ of $(\frac{1}{4} \div \frac{1}{5})$; $(\frac{1}{2} \div \frac{1}{3}$ of $\frac{1}{4}) \div \frac{1}{5}$.

35. Give instances of incorrect cancelling; also simplify, (cancelling when possible,)

$$\frac{1\frac{1}{3}}{1\frac{2}{3}+2\frac{1}{3}}; \quad \frac{1\frac{1}{3}+\frac{1}{2}}{1\frac{1}{3}-\frac{1}{2}}; \quad \frac{1\frac{1}{2}\times\frac{1}{2}}{1\frac{1}{2}\div\frac{1}{2}}; \quad \frac{\frac{1}{8}+\frac{1}{7}}{\frac{1}{8}-\frac{1}{7}}; \quad \frac{\frac{1}{8}\times\frac{1}{7}}{\frac{1}{8}\div\frac{1}{7}}$$

36. Subtract $11\frac{3}{11}$ inches from 11 feet $0\frac{3}{16}$ inch.

37. Multiply 3 hours $3\frac{7}{160}$ mins. by 270, and prove the result (i) by dividing by factors, (ii) by Long Division.

38. What are the rules for multiplying and dividing by Fractions?

Divide £8 0s. $7\frac{6}{25}$d. by $10\frac{1}{5}$, and prove the result by multiplication.

39. Express 8 as a fraction of 9; and $8\frac{1}{2}$ half-crowns as a fraction of $2\frac{1}{8}$ guineas.

DECIMALS.

40. .3 is an abbreviation for 3÷10, *or* $\frac{3}{10}$.
11.3 ,, ,, 113÷10, *or* $\frac{113}{10}$, *or* $11\frac{3}{10}$.
.13 ,, ,, 13÷100, *or* $\frac{13}{100}$.
.03 ,, ,, 3÷100, *or* $\frac{3}{100}$.
1.13 ,, ,, 113÷100, *or* $\frac{113}{100}$, *or* $1\frac{13}{100}$.

Write out the above five lines (i) changing 3, whenever it occurs, into 7; (ii) changing it into 9.

Example.—Express as a decimal 813÷100, *or as we may write it* $\frac{813}{100}$, or $8\frac{13}{100}$.

As there are *two* 0's in 100 we write down 813, and then mark off the *two* last figures. *Ans.* 8.13.

Observe: We always mark off as many figures as there are 0's in the power of 10 which is the Denominator of the corresponding vulgar fraction.

Example.—Express as a decimal "13 divided by 10000," or its equivalent $\frac{13}{10000}$.

We must mark off 4 figures, since there are 4 0's in 10000.

But 13 consists only of 2 figures;
so place two 0's to the left of the 13. *Ans.* .0013.

Warning: Never leave out the Decimal Point.

Ex. 40. A.—Express as decimals

4÷10; 114÷10; 14÷100; 114÷100;

4÷100. What must be placed here before the 4 so as to enable us to mark off *two* places?

8÷1000; 118÷10000; 1111÷10;

$\frac{7}{10}$; $\frac{7}{100}$; $\frac{17}{1000}$; $1\frac{13}{100}$; $1\frac{3}{1000}$.

Ex. 40. B.—Express as Vulgar Fractions

.3, .13, 1.4, 51.114,
1.4, 2.14, .04, 3.002,
4.003, 3.0084, 11.00061.

Specimen Example.—121.00121.

121.00121 means "12100121 divided by 100,000,"
or, which is the same thing,
"121 (a whole number) ; *and* 121 divided by 100,000."

Ans. $\frac{12100121}{100000}$ *or* $121\frac{121}{100000}$.

It is well at first to write out answers in *both* of the above forms.

Observe: We have here 100,000 (that is 1 followed by *five* 0's) because there are *five* figures after the decimal point in 121.00121.

41. To express any Vulgar Fraction as a Decimal.

General Rule.—Place a decimal point, followed by 0's, at the right of the Numerator, then divide it by the Denominator.

Example.—Bring $\frac{1}{16}$ to a decimal.

We write down 1.0000 &c. and divide it by 16, thus

```
16)1.00 &c.(.0625
     96
     ——
      40
      38
      ——
       80
       80          Ans. .0625.
```

"16 in 1" will not go,
so we cross over the point in the dividend,
and thereupon place a point in the quotient.
"16 in 10" will not go,
so we place a 0 after the point in the quotient.
We now have "16 in 100" and proceed as in ordinary Long Division.

We supply in the dividend as many 0's only as are needed.

The directions below the example just given must be carefully read, for it is most important to know where the point is to stand in the result.

Ex. 41. A.—Bring to Decimals

$\frac{1}{20}, \frac{1}{4}, \frac{1}{8}, \frac{3}{16}, \frac{2}{15}, \frac{3}{20}, \frac{13}{20}, \frac{17}{40}, \frac{7}{80}$.

Example.—Bring to a Decimal $\frac{2}{3}$.

Divide the Numerator by the Denominator as before.

$$\begin{array}{r} 3\underline{)2.0000,\ \&c.} \\ .6666,\ \&c. \end{array}$$

Thus we have a never-ending succession of 6's
which we express thus $.\dot{6}$.

Such Decimals are called *recurring* or *circulating*.

Ex. 41. B.—Express as Decimals $\frac{1}{3}, \frac{7}{9}, \frac{4}{9}, \frac{8}{9}$.

Example.—Bring to a decimal $\frac{1037}{3330}$.

3330)1037.00 &c.(.3114
9990
3800 (Observe this remainder.)
3330
4700
3330
13700
13320
380 (Observe this remainder.)

Thus at last we get the same remainder as one which we have had before; so that if we continue the division we shall have the same figures all over again, and the quotient will be

.3114114114, &c. and so on for ever,

which, for the sake of shortness, we express thus

$$.3\dot{1}1\dot{4}.$$

114 is called the recurring or circulating *period*.

The two dots are placed over the first and last figures of the *period*.

Such Decimals as $3\dot{1}1\dot{4}$ are called Mixed Circulators because, besides the recurring part, there is the figure 3, which does not recur.

Decimals which do not recur are called Terminating Decimals.

Ex. 41. C.—(i.) Express in the abbreviated forms

.5882882882, &c.; .808808808, &c.; .345346346, &c.

(ii.) Write at length (as just above) the decimals expressed by $.\dot{9}19\dot{9}$; $.9\dot{1}9\dot{9}$; $.919\dot{9}$.

(iii.) Express as Decimals

$\frac{1}{6}$; $\frac{5}{6}$; $\frac{7}{60}$; $\frac{8}{75}$; $\frac{10}{11}$; $\frac{61}{495}$; $\frac{217}{330}$.

42. Reason for the Rule for reducing a Vulgar Fraction to a decimal.

Take the fraction $\frac{7}{8}$:

We wish to express $\frac{7}{8}$ as a fraction *whose denominator is a power of* 10;

For when this is done, we can at once turn it into a decimal, *by writing it in the abbreviated form as explained above,* (40.)

Thus we know that $\frac{1}{4} = \frac{25}{100}$,

We have then only to write $\frac{25}{100}$ in the abbreviated form .25,

And $\frac{1}{4}$ is expressed as a decimal—*viz.* .25.

To return to $\frac{7}{8}$:

We cannot express it as a fraction whose denominator is 10 or 100,

Because neither 10 *nor* 100 *contains* 8,

But we can express it as a fraction whose denominator is 1000, *because* 1000 *contains* 8.

Now the rule for finding the new numerator tells us to

Divide 1000 by 8, and then multiply by 7;

or to multiply 1000 by 7 and divide by 8,

that is, to divide 7000 *by* 8,

$$\begin{array}{r} 8\underline{)7000} \\ 875 \end{array}$$

thus it is, by dividing the numerator 7 (followed by 0's) by the denominator 8, that we get the *new* numerator, from which we can pass at once to the decimal form. We have accordingly,

$$\frac{7}{8} = \frac{875}{1000} = .875.$$

Ex. 42.—Express as decimals, as in the Specimen Example below,

$$\frac{1}{2},\ \frac{3}{4},\ 3\frac{3}{8},\ 4\frac{2}{5},\ \frac{8}{25},\ 1\frac{7}{250},\ \frac{9}{125},\ 2\frac{19}{40}.$$

Specimen Example.—$3\frac{11}{125}$.

What is the least power of 10 which contains 125? 1000.

We must then express $\frac{11}{125}$ as a fraction whose denominator is 1000,

$$\text{thus, } 3\tfrac{11}{125}, = 3\tfrac{88}{1000} = 3.088.$$

Ans. 3.088.

43. What Vulgar Fractions can be brought to *Terminating* Decimals ?

If there is no power of 10 which *contains* the denominator of the fraction,

The fraction cannot be expressed as another fraction whose denominator is some power of 10;

It cannot therefore be expressed as a terminating decimal.

The factors of 10 are 2 and 5;

of 1000 are 2, 2, 2, 5, 5, and 5.

1000 therefore *contains* any number which is the product of not more than three 2's, and not more than three 3's.

We may go on to say, that any number whose factors are only 2's and 5's, is contained in a sufficiently high power of 10,

thus, (for instance,) $2^7 \times 5^8$ or 50,000,000

is contained in 10^8 or 100,000,000.

Thus, any Vulgar Fraction whose denominator contains only powers of 2 and of 5, *and no other factor*, can be expressed as a fraction whose denominator is some power of 10,

and so can be expressed as a terminating decimal.

Observe: The lowest power of 10 which contains $2^7 \times 5^8$ is the *eighth*, the same as the highest power of either of its prime factors.

This shows us that the number of decimal places in the corresponding decimal will be also 8,

being the same as the number of 0's in the denominator which is not expressed.

Warning: The fraction must always be brought to its Lowest Terms, before the factors of the denominator are examined.

Ex. 43.—Ascertain which of the following can be expressed as *Terminating* Decimals, and how many Decimal places (or figures after the point) there will be :—

$\frac{1}{4}, \frac{3}{25}, \frac{9}{50}, \frac{9}{51}, \frac{11}{64}, \frac{13}{65}, \frac{119}{120}, \frac{119}{125}.$

Specimen Examples.—

(i) $3)\frac{9}{51}=\frac{3}{17}$, (bringing to Lowest Terms.)

Here the Denominator *is*, and so *contains*, the simple factor, 17, which is neither 2 nor 5.

$\therefore$ $\frac{9}{51}$ cannot be expressed as a Terminating Decimal.

(ii) $\frac{17}{640}$.

Here $640=2\times2\times2\times2\times2\times2\times2\times5=2^7\times5$.

As 640 contains no factor except 2's and a 5, $\frac{17}{640}$ can be expressed as a Terminating Decimal; and the number of Decimal places will be 7, since 640 contains 2^7, &c.

When the fraction is in its Lowest Terms, the Numerator has nothing to do with the question.

44. The following facts should now be understood and remembered :—

(i) 0 at the right of a Decimal makes no difference in its value, for $.41=\frac{41}{100}=\frac{410}{1000}=.410$.

In the Answers 0's to the right of Decimals should be omitted.

(ii) To multiply a Decimal by 10
the point should be moved one place to the right.

$$3.24\times10=\frac{324}{100}\times10=\frac{324}{10}=32.4.$$

(iii) To divide a Decimal by 10
the point should be moved one place to the left,

$$3.24\div10=\frac{324}{100}\div10=\frac{324}{1000}=.324.$$

(iv) Such a Decimal as .13 is divided by 10 by inserting 0, thus, $.13 \div 10 = .013$.

(v) As multiplying or dividing by 10 is accomplished by moving the point one place to the right or left so multiplying or dividing by 100 is accomplished by moving the point two places, &c.

Example.—Divide $1.0\dot{9}$ by 100. *Ans.* $.010\dot{9}$.

Example.—Multiply $1.0\dot{9}$ by 1000. *Ans.* $1099.\dot{9}$.
(Remembering that $1.0\dot{9}$ is an abbreviation for 1.09999, &c.)

Example.—Multiply .003 by 10000. *Ans.* 30.

Observe : **30 being a whole number, the 0's to the left of the 3 are omitted as useless.**

Ex. 44.—Multiply $.7$; $.07$; $.0\dot{7}$; $.\dot{0}\dot{7}$, severally by 10, 100, 1000;
also divide them severally by 10, 100, 1000.
Multiply $.012$ by 10 and by 1000.
Multiply $.12\dot{3}$ by 100 and by 1000.
Divide 14.01 by 100 and by 10000.
Divide $5.\dot{4}\dot{5}$ by 100 and by 10000.

QUESTIONS ON STEPS 40—44.

40. For what is 1.17 an abbreviation ?

Express as a Decimal "3 divided by 1000."

41. Write down any recurring Decimal, saying whether or not it is a mixed circulator.

Write 3.334333433343334 &c. in the abbreviated form.

Bring $\frac{3}{70}$ to a Decimal.

42. How may we most simply convert $\frac{1}{4}$ into a Decimal ?

Express $\frac{3}{8}$ as a fraction whose Denominator is some power of 10, and thence express it as a Decimal.

43. What factors only must the Denominator of a fraction —in its Lowest Terms—contain, if it is to be expressed as a terminating Decimal ?

How may we foresee the number of Decimal places ?

Can $\frac{12}{375}$ be expressed as a terminating Decimal ? How many Decimal places will there be in $\frac{3}{320}$?

44. Give the rules for multiplying or dividing Decimals by 10, 100, &c.

Shew that .43=.430.

Multiply $8.0\dot{5}$ by 100, and divide $131.\dot{4}13\dot{1}$ by 10000.

45. To bring a Recurring Decimal to a Vulgar Fraction.

Ex.—Bring to Vulgar Fractions (i) $.\dot{3}1\dot{7}$; and (ii) $.012\dot{3}\dot{4}$.

Write down the Decimals as given, omitting the points and the dots, thus 317, 1234.

Now, in each case, to find the *Numerator*, subtract *from the whole*, the figures (if any) which do not recur.

317

Remains unchanged,
since all three figures recur;

$$\begin{array}{rr} \text{but} & 1234 \\ \text{when we subtract} & 12 \\ \hline \text{Becomes} & 1222 \end{array}$$

Warning: Do not subtract thus $\begin{array}{r} 34 \\ 12 \\ \hline 22 \end{array}$

To find the Denominator, write as many 9's as there are circulating figures in the Decimal, followed by as many 0's as there are figures which do not circulate.

Thus we have here (i) $\frac{317}{999}$; (ii) $\frac{1222}{99000}$.

Now reduce (if possible) to Lower Terms $2\overline{)\frac{1222}{99000}}=\frac{611}{49500}$.

Ans. (i) $\frac{317}{999}$; (ii) $\frac{611}{49500}$.

Ex. 45.—Bring to Vulgar Fractions

$.0\dot{1}$; $.1\dot{2}$; $.\dot{2}1\dot{7}$; $.\dot{3}1\dot{6}$; $.\dot{1}$; $.\dot{1}\dot{2}$;
$3.\dot{0}\dot{3}$; $.1\dot{3}$; $.012\dot{3}\dot{6}$; $.12\dot{3}\dot{6}$; $.0\dot{1}$; $.12\dot{3}$;
$5.17\dot{8}\dot{0}$; $.01\dot{2}$; $.08\dot{9}$; $.0\dot{8}\dot{9}$; $.\dot{0}8\dot{9}$.

46. Addition and Subtraction of Decimals.

Example.—Simplify $2+.13-1.3+.01\dot{3}-4.\dot{4}\dot{1}+6.\dot{0}0\dot{1}$.

1*st Step.*—Add together those terms which *are not* preceded by the – sign, thus

$$\begin{array}{rll}
2. & & \\
 & 13 & \\
 & 01333333 & \&c. \\
6. & 00100100 & \&c. \\
\hline
8. & 14433433 & \&c.
\end{array}$$

Observe: (i.) A point is placed at the right of the whole number 2.

(ii.) The decimal points are placed exactly underneath one another in a straight line.

(iii.) The circulating figures of the recurring decimals are written over-and-over again, so as to supply a good many figures *beyond the terminating decimals.*

2*nd Step.*—Add together in the same way the terms which *are* preceded by the – sign, thus

$$\begin{array}{ll}
1.3 & \\
4.41414141 & \&c. \\
\hline
5.71414141 & \&c.
\end{array}$$

Observe: We take care to have the same number of figures to the right of the point *here* as in the former case.

3*rd Step.*—Subtract the latter result from the former, thus

$$\begin{array}{ll}
8.14433433 & \&c. \\
5.71414141 & \&c. \\
\hline
2.43019292 & \&c.
\end{array}$$

We have thus found the result, but we cannot be sure that the last one or two figures are *correct,* we write therefore

"*Ans. correct to* 6 *places of decimals,* 2.430192."

Observe: We may find the complete answer by taking more figures, thus,

$$\begin{array}{ll}
8.144334334334334 & \&c. \\
5.714141414141414 & \&c. \\
\hline
2.430192920192920 & \&c.
\end{array}$$

In this result we can see what the recurring figures really are, and that the true answer is $2.43\dot{0}1929\dot{2}$.

We see too that the former result was correct as far as it went.

Example.—Find the value of $3.34567-.81\dot{3}-1.\dot{1}$ correct to five places of decimals.

Here we will write out our recurring decimals so as to get 7 places of decimals, 2 more than the number asked for, that being generally enough

$$\begin{array}{lr} \text{we add} & .8133333 \\ \text{and} & 1.1111111 \\ \hline & 1.9244444 \end{array}$$

$$\begin{array}{rr} \text{And now from} & 3.3456700 \\ \text{we subtract} & 1.9244444 \\ \hline & 1.4212256 \end{array}$$

(two 0's being appended, to give enough places of decimals.)

Now omit the 56, which may not be really correct owing to the omission of figures which should stand at the right of those taken, we have *Answer correct to* 5 *places* 1.42122.

The two examples just given should be worked through without the Book, before going to the following Exercise.

Ex. 46.—Find the value of

$$.3\dot{1}\dot{1}-4.1+8.8997-.04\dot{7};$$

$$.8184+12+.\dot{5}\dot{7}-8.00\dot{5}\dot{7};$$

$$108+1.08+.0108-.\dot{1}0\dot{8}.$$

Warning: Do not forget to place points at the right of the whole numbers. The recurring decimals here should be written out to 8 places of decimals, but it will be seen that it is often not necessary to go so far.

$$.3\dot{1}+.1\dot{3}; \quad .3\dot{1}-.1\dot{3};$$

$$.\dot{4}\dot{8}-.\dot{4}\dot{8}; \quad .\dot{8}\dot{4}+.84;$$

$$1.9-.3\dot{2}; \quad 1.3\dot{2}-.99.$$

The last 6 should be *proved* by reducing the decimals to vulgar fractions, adding or subtracting, and reducing the result to a decimal.

47. Multiplication of Terminating Decimals.

Multiply as in whole numbers,

then mark off as many decimal places as there are in the multiplier and multiplicand together.

Example.—Multiply .031 by .0013.

$$\begin{array}{r} 31 \\ 13 \\ \hline 93 \\ 31 \\ \hline 403 \end{array}$$

Ans. .0000403.

Observe: There are 3 decimal places in .031
and 4 in .0013
accordingly we must have 7 decimal places in the product.
As then 403 consists of only 3 figures,
we must prefix four 0's, thus, *Ans.* .0000403.

Ex. 47. A.—Multiply .041 by .0014; 3.1 by .014;
.0081 by .123; 10.9 by 71.

Division of Terminating Decimals.

Example.—Divide .0052013 by .000013.

Move the decimal point to the right of the divisor thus making .000013 become 13.

And move the decimal point the same number of places, *viz.* 6, to the right *in the dividend,*

thus making .0052013 become 5201.3.

Now divide as in whole numbers,

placing a point in the quotient *when we cross the point in the dividend.*

13)5201.3(400.1
52
01.3

Ans. 400.1.

Ex. 47. B.—Divide .0039013 by .000013.

Multiply 81.0031 by .04, and by .004, and prove the results.

Divide 81.0032 by .4; by .04; by .0004; by .16;
and by 160.

Observe: When the divisor is a whole number, the position of the point in the dividend has not to be changed.

In dividing we may have to supply 0's in the dividend, at the right of the figures given, as we do in bringing vulgar fractions to decimals.

Ex. 47. C.—Simplify $\frac{3.36}{.7}$; $\frac{62.3}{.04}$; $\frac{.57}{6}$; $\frac{80}{.18}$.

Specimen Example.—Simplify $\frac{.2}{.015}$

$\frac{.2}{.015} = .2 \div .015.$

$= 200 \div 15$, moving the point to the right of the divisor, and therefore also 3 places to the right in the dividend.

$= 40 \div 3$, cancelling 5.

3)40.000 &c.
13.333 &c. *Ans.* $13.\dot{3}$.

The same methods may be used sometimes for multiplying and dividing recurring decimals, when the multiplier or divisor is a whole number or a terminating decimal.

Example.—Multiply $.\dot{1}\dot{3}$ by .4.

131313 &c.
4
525252 &c. *Ans.* $.0\dot{5}\dot{2}$.

Example.—Prove the last result by division.

$.0\dot{5}\dot{2} \div .4 = .\dot{5}\dot{2} \div 4.$

4).525252 &c.
.131313 &c. *Ans.* $.\dot{1}\dot{3}$.

Ex. 47. D.—Multiply $.0\dot{3}$ by $.5$; $.1\dot{2}\dot{3}$ by $.2$;
$.\dot{1}2\dot{3}$ by $.07$; $.\dot{2}$ by 13;
and prove each result by division.

48. When we wish to multiply or divide recurring decimals, the best plan, as a rule, is to bring them to vulgar fractions, then to multiply or divide as the case may be, and then bring back our result to a decimal.

Example.—Simplify $.1\dot{3}\times.\dot{7}\div.1\dot{5}$.

$$\begin{aligned}.1\dot{3}\times.\dot{7}\div.1\dot{5}&=\tfrac{12}{90}\times\tfrac{7}{9}\div\tfrac{14}{90},\\&=\tfrac{12}{90}\times\tfrac{7}{9}\times\tfrac{90}{14}=\tfrac{2}{3},\end{aligned}$$

$$\begin{array}{r}3\underline{)2.000\ \ \&c.}\\ .666\ \ \&c.\end{array}\qquad \textit{Ans. } .\dot{6}.$$

Ex. 48. A.—Simplify $.1\dot{4}\div.2\dot{8}$; $.3\times.\dot{3}\div.9$;
$1.4\div1.\dot{4}$; $.8\times.\dot{5}\div1.\dot{7}$.

Example.—Simplify $.1\dot{7}\times\cdot\frac{3}{4}$
(i) by reducing to vulgar fractions;
(ii) without so reducing.

(i) $.1\dot{7}\times\cdot\tfrac{3}{4}=\tfrac{16}{90}\times\dfrac{\frac{3}{10}}{4}=\tfrac{1}{75}=.01\dot{3}.$

(ii)
$$\begin{array}{r}.17777\ \ \&c.\\ \underline{.3}\\ 4\underline{).053331}\\ .01333\ \ \&c.\end{array}$$

observe the 1 here is not *correct*: it results from the omissions of the next 7 in the multiplicand, so in dividing we do not take it into account.

Ans. $.01\dot{3}$ *or* $\frac{12}{900}$ *or* $\frac{1}{75}$.

Ex. 48. B.—Simplify each of the following :—

(i) by reducing to vulgar fractions ;
(ii) without reducing,

and shew that the results agree :

$$1.\dot{2} \div \tfrac{11}{12}\,;\ .1\dot{7} \times \cdot\tfrac{4}{3}\,;\ .\dot{1}\dot{9} \times \tfrac{3}{4}\,;\ .00\dot{3} \div \tfrac{\cdot 2}{11}.$$

The following facts should be remembered :—

$$.25 = \tfrac{1}{4}\,;\ .5 = \tfrac{1}{2}\,;\ .75 = \tfrac{3}{4}\,;$$
$$.\dot{3} = \tfrac{3}{9} = \tfrac{1}{3}\,;\ .\dot{6} = \tfrac{6}{9} = \tfrac{2}{3}\,;$$
$$.\dot{9} = \tfrac{9}{9} = 1.$$

Recurring decimals in which the period is 9 may be expressed as terminating decimals.

Thus $.1\dot{9} = \frac{18}{90} = \frac{1}{5} = .2$; $.2\dot{9} = \frac{27}{90} = \frac{3}{10} = .3$.

Similarly $.40\dot{9} = \frac{409-40}{900} = \frac{369}{900} = \frac{41}{100} = .41$.

Observe : By the last three results, that $\dot{9}$ at the end of the given decimals may be omitted if we increase the preceding figure by 1,

thus $.3\dot{9} = .4$; $.17\dot{9} = .18$ &c.

Ex. 48. C.—Express in Simpler Forms

$$.5\dot{9}\,;\ .30\dot{9}\,;\ .14\dot{9}\,;\ .8\dot{9}\,;\ .000\dot{9}\,;$$
$$\tfrac{1}{3} \text{ of } .20\dot{9} + \tfrac{1}{4} \text{ of } .30\dot{9} + \tfrac{1}{5} \text{ of } .40\dot{9}.$$

49. It is now time for some explanation of the reason of the rule for reducing recurring decimals to vulgar fractions.

Take, as an example, the decimal $.512\dot{3}\dot{4}$.

Let us find what is the value of $\frac{99}{100}$ of this decimal.

Observe : We take $\frac{99}{100}$ when, as in this example, *two* figures recur ; if *three* figures recurred we should take $\frac{999}{1000}$; if only *one* figure, $\frac{9}{10}$, &c.

To find then $\frac{99}{100}$ of this decimal, *we subtract its hundredth part, so that* 99 *hundredths shall remain.*

The hundredth part of .5123434 &c. is .005123434 &c.

$$\begin{array}{rl} \text{Thus we have} & .5123434\ \&c. \\ & \underline{.0051234\ \&c.} \\ & .5072200\ \&c. = .50722. \end{array}$$

We see that $\frac{99}{100}$ of our *recurring* decimal turns out to be the *terminating* decimal .50722; what then is the *whole* value of the recurring decimal?

This is an example in Rule of Three, and may be done thus

$$\begin{array}{lcl} \frac{99}{100} \text{ of the decimal is } .50722 \textit{ or } \frac{50722}{1000000}, \\ \therefore \frac{1}{100} \quad \text{,,} \quad \text{is } \frac{1}{99} \text{ of } \frac{50722}{1000000}, \\ \therefore \text{the whole decimal is } \frac{100}{99} \text{ of } \frac{50722}{1000000} \textit{ or } \frac{50722}{99000}. \end{array}$$

Here then we have the very result which we should get by the ordinary rule for reducing a recurring decimal to a vulgar fraction.

When then we write down the decimal to the end of the first period and subtract the figures which do not circulate,

we are really, supposing that the *period* consists of two figures, *subtracting from the decimal its hundredth part,* or finding $\frac{99}{100}$ of the whole decimal.

So that multiplying what we thus find by $\frac{100}{99}$ we have the value of the whole decimal.

Thus we see how it is that the Denominator of the equivalent vulgar fraction must begin with two 9's.

Ex. 49. A.—Find by the method just given the values of the recurring decimals

$$.1\dot{7};\ .6\dot{1}\dot{7};\ .23\dot{4}\dot{5};\ .1\dot{2}3\dot{4};$$
$$.13\dot{4};\ .1\dot{3}\dot{4};\ .0\dot{1}3\dot{4};\ .\dot{0}13\dot{4};$$
$$.12\dot{1}\dot{3};\ .120\dot{1}\dot{3};\ .0\dot{1}\dot{3};\ .0\dot{1}\dot{3}.$$

Specimen Example.—$.03\dot{4}5\dot{6}$. See next page.

To find $\frac{999}{1000}$ of $.03\dot{4}5\dot{6}$,

$$\begin{array}{rl} \text{We subtract from} & .03456456456 \text{ \&c.} \\ \text{its thousandth part} & \underline{.00003456456 \text{ \&c.}} \\ & .03453000 \text{ \&c.} \end{array}$$

Thus $\frac{999}{1000}$ of our decimal is .03453 *or* $\frac{3453}{100000}$,
What then must be its whole value ?

Ans. $\frac{3453}{100000} \times \frac{1000}{999}$ *or* $\frac{3453}{99900}$.

A fraction can easily be brought to a recurring decimal when, by multiplying both Numerator and Denominator by any small number, it can be brought to a form which may be recognised as equivalent to some recurring decimal, thus

$$\frac{3}{45} = \frac{6}{90} = .0\dot{6}\,; \quad \frac{13}{45} = \frac{26}{90} = \frac{28-2}{90} = .2\dot{8}\,;$$

$$\frac{17}{495} = \frac{34}{990} = .0\dot{3}\dot{4}\,; \quad \frac{217}{495} = \frac{434}{990} = \frac{438-4}{990} = .4\dot{3}\dot{8}.$$

Ex. 49. B.—Express as recurring decimals

$\frac{8}{45}$; $\frac{17}{45}$; $\frac{7}{30}$; $\frac{14}{15}$; $\frac{8}{11}$; $\frac{19}{110}$; $\frac{119}{330}$.

Each result may be proved by the ordinary method.

50. Reduction.

Find the value of

.125 of £1,

or, as it may be expressed £.125.

This is merely an example in reduction.

The decimal point makes no difference in the ordinary rule for reducing.

We are to bring .125 of £1 *or* £.125 to shillings and pence.
To reduce pounds to shillings and pence
we multiply by 20, and then by 12.

It is the same here

$$\begin{array}{r} £.125 \\ 20 \\ \hline 2.500\text{s.} \end{array}$$

Thus £.125=2.5 shillings.

We do not wish to reduce the 2 whole shillings any further but .5 of a shilling must be brought to pence in the usual way.

$$\begin{array}{r} .5\text{s.} \\ 12 \\ \hline 6.0\text{d.} \end{array} \qquad \textit{Ans. } 2\text{s. } 6\text{d.}$$

Ex. 50. A.—Find the value of

.5 of 1 yard; .375 of £1; .25 of 1 shilling;
.05 of £1; .75 of 1 yard; .175 of a ton.

Example.—Find the value of $3.1\dot{3}$ of 2 sq. yards 4½ sq. ft.

Here 2 sq. yds. 4½ sq. ft.=22½ sq. ft.

We have then to reduce $3.1\dot{3}$ areas, each consisting of 22½ sq. ft. to sq. yds. and sq. ft.

We first multiply 22½ sq. ft. by the whole number 3.

22½ sq. ft. × 3 = 67½ sq. ft.

We then multiply 22½ sq. ft. by $.1\dot{3}$.

$22\tfrac{1}{2}$ sq. ft. $\times .1\dot{3} = \frac{45}{2}$ sq. ft. $\times \frac{12}{90} = 3$ sq. ft.,

adding, we have 70½ sq. ft. *or* 7 sq. yds. 7 sq. ft. 72 sq. in.

Ex. 50. B.—Find the value of $4.1\dot{4}$ of 2 sq. yds. 4½ sq. ft.;
$2.0\dot{3}$ of 16s. 6d.; $2.\dot{0}\dot{3}$ of 4s. 1½d.;
$8.2\dot{3}$ of £1 1s. 8d.; $3.8\dot{1}$ of an hour.

Ans. 10sq. yds. 3¼sq. ft.; 8s. 4½d.; £1 13s. 6d.; £14 8s. 2d.; 3h. 48m. 40s.

Simplify

$.\dot{5}$ of $\frac{1}{.3}$ of .6 ft. + $.1\dot{9}$ of $.1\dot{7}$ ft. − .18 of 1 yd. 1 ft. 3 in.

Here $.\dot{5}$ of $\frac{1}{.3}$ of .6 ft.

$=\frac{5}{9}$ of $\frac{1}{\frac{3}{10}}$ of $\frac{6}{10}$ of 1 ft.

$=\frac{10}{9}$ of 1 ft. $=1.\dot{1}$ of a foot.

$.1\dot{9}$ of $.1\dot{7}$ ft.

$=\frac{18}{90}$ of $\frac{16}{90}$ of 1 foot.

$=\frac{8}{225}$ of 1 foot $=.03\dot{5}$ of a foot.

.18 of 1 yd. 1 ft. 3 in.

$=.18$ of $4\frac{1}{4}$ ft.

$=.18$ of $\frac{17}{4}$ ft. $=.9$ of $\frac{17}{2}$ ft. $=\frac{1.53}{2}$ ft. $=.715$ of 1 ft.

Adding	1.1111 &c.	from	1.14666 &c.
to	.0355 &c.	substracting	.71500
we have	1.1466 &c.	we have	.43166 &c.

Ans. $.431\dot{6}$ of a foot, *or* (multiplying by12) 5.18 inches.

Ex. 50. C.—Simplify

$1.1\dot{4}$ of 14s. − $2.\dot{3}$ of a crown + $3\frac{3}{4}$ of £.3;

3.7 of £$3.\dot{7}$ + $7.\dot{3}$s. − $.7\dot{3}$ of 5s. 6d.;

.443 of 8 yds. − .043 of a fur. + 33.3 of $.\dot{8}$ yds.

Answers. $11.8\dot{5}$ shillings; £14 $2.8\dot{4}$ shillings; 23.684 yds.

Express 7lbs. as the decimal of 3 tons.

This is another example in reduction.

To express 7000000lbs. (suppose) in tons we should divide by 28, 4, and 20.

To express 7lbs. in tons *we do the same,* not however expecting to obtain a result as *whole* tons, but merely as the decimal of 1 ton.

28)7.000 &c. *lbs.*
4) .250 &c. *qrs.*
20).06250 &c. *cwts.*
3).0031250 &c. *tons* dividing by 3 to bring ton weights to 3-ton wts.
.00104166 &c.

Ans. $.001041\dot{6}$ *of* 3 *tons.*

Ex. 50. D.—Express 14lbs as the decimal of 4 tons;
6d. „ „ £2;
9d. „ „ £3;
3 days „ „ 2 weeks.

Answers. .0015625; .0125; .0125; $.\dot{2}14285\dot{7}$.

Reduce 2cwts. 3qrs. 7lbs. to the decimal of 4 tons.

1*st Step.—Reduce 7 lbs. to the decimal of* 1qr.

dividing 7 by 28, we have .25qrs.

adding this decimal to the 3qrs. contained in the question we have as our whole amount

2cwts. 3.25qrs.

2nd Step.—Reduce 3.25qrs. *to the decimal of* 1cwt.

dividing 3.25 by 4 we have .8125cwt.

adding this decimal to the 2cwts. contained in the question we have

2.8125cwts.

3rd Step.—20)2.8125 *cwts.*
4) .140625 *of* 1 *ton*
.03514625 *of* 4 *tons.*

Ans. .03514625.

Ex. 50. E.—Reduce 3 cwts. 3 qrs. 14 ℔s. to the decimal of 5 tons;

3s. 4¼d. to the decimal of £2;

6s. 8½d. „ „ £1 10s.

here reduce to decimal of half-a-sovereign, and then to decimal of 30s.

1yd. 1ft. 1in. to the decimal of 1 furlong.

Answers. .635; .083854; $.2236\dot{1}$; $.0016409\dot{5}50\dot{0}$.

Express .625 of 7s. 6d. as the decimal of 5s. 3d.

Suppose the question to be

Reduce 625 coins, each worth 7s. 6d., to other coins, each worth 5s. 3d.

We should reduce the 625 seven-and-sixpences to pence
by multiplying by 90,
and then reduce back to five-and-threepences
by dividing by 63;
following the same method with the decimal

we have $.625 \times \frac{90}{63}$,

$= .625 \times \frac{10}{7}$,

$= 6.25 \div 7$,

$$\begin{array}{l} 7\overline{)6.2500}\ \&c. \\ \hline .89\dot{4}285\dot{7}\dot{1} \end{array}$$

Ans. $.89\dot{4}28571\dot{}$.

Ex. 50. F.—Express .325 of 2s. 6d. as the decimal of 3s.;

1.55 of 4s. as the decimal of 7s. 9d.;

.06 of 4 days, as the decimal of 3 weeks;

.001 of 10 guineas as the decimal of 5s. 3d.;

$1.\dot{5}$ of 18s. as the decimal of £1 18s.

Answers. $.2708\dot{3}$; .8; $.1\dot{1}42857\dot{}$; .04; .5.

Express $2.1\dot{3}$ of 1cwt. 1qr.
as the decimal of $.1\dot{5}$ of 3cwts. 2qrs.

This example involves
multiplication and division of recurring decimals.

Reducing therefore to vulgar fractions, we have

$2\frac{12}{90}$ of 5qrs. to decimal of $\frac{15}{90}$ of 14qrs.

or $\frac{32}{15}$ of 5qrs. to the decimal of $\frac{7}{3}$ qrs.

or $\frac{32}{3}$ qrs. to the decimal of $\frac{7}{3}$ qrs.

We write down then $\frac{32}{3} \div \frac{7}{3}$ *or* $\frac{64}{21}$, and reduce to a decimal.

Ans. $4.\dot{5}7142\dot{8}$.

Ex. 50. G.—Express $.\dot{3}$ of £4 as the decimal of $.\dot{8}$ of £1 5s ;
.2 of $.\dot{5}$ of 6s. ,, 1.5 of 2s. 6d. ;
.35 of $10\frac{1}{2}$ days ,, .7 of 3 weeks.

Answers. 1.2 ; $.1\dot{7}$; 2.5.

Observe : All the former Examples in (50) may be proved by reducing the decimals *at once* to Vulgar Fractions, (as is done on this page just above), and finally reducing the result to a decimal.

QUESTIONS ON STEPS 45—50.

45. In bringing a recurring Decimal to a Vulgar Fraction, how are the Numerator and Denominator respectively found?

Bring to Vulgar Fractions,

$.045\dot{6}$; $.04\dot{5}\dot{6}$; $.0\dot{4}5\dot{6}$; $.\dot{0}45\dot{6}$.

46. Simplify $1.045\dot{6}-2.04\dot{5}\dot{6}+3.0\dot{4}5\dot{6}-.\dot{0}45\dot{6}$; and prove the result by Vulgar Fractions.

47. 48. Multiply .051 by .3, and divide the result by 1.7.

Simplify $.01\dot{7} \div 3.2$ and $.\dot{7}1\dot{0} \times 11.1$. both by Decimals and by Vulgar Fractions.

Simplify $\frac{.4\dot{9}}{5} + \frac{.3\dot{9}}{2} - .2\dot{9}$.

49. (i) Find the value of $\frac{9}{10}$ of $.4\dot{7}$; and thence shew that the whole value of the decimal is $\frac{43}{90}$.

Find the value of $\frac{99}{100}$ of $.2\dot{3}\dot{5}$; and thence express the whole decimal as a Vulgar Fraction.

(ii) Express as recurring decimals,

$\frac{8}{225}$; $\frac{88}{225}$; $\frac{7}{111}$; $\frac{77}{111}$.

50. (i) Reduce .018 of £5 to shillings and pence.

(ii) Find the value of

$.\dot{3}$ of an inch + $.0\dot{3}$ of a foot + $.00\dot{3}$ of a yard.

(iii) Reduce $.4\dot{1}$ of £1 7s. to the decimal of £1 17s.

12 SHORT EXAMINATION PAPERS ON THE 50 STEPS.

I. 1. What numbers are represented by
3^2, 3^3, 3^4, $3^2 \times 2^3$, $2^3 \times 5^2 \times 7$, $4^2 + 5^2$, $4^2 \times 5^2$, $2^4 + 2^5$, $2^4 \times 2^5$?

2. Write down all the numbers of which 48 is a multiple.

3. Write down five numbers of which 12 is a measure.

4. Shew what numbers, less than 12, are measures of 12321, 32130, and 31913;

Also, write down any number of which 8, 9, and 11 are measures, and shew that they are so.

5. Simplify $3-4+5-6+7$ and $3^5-4^4+5^3-6^2+7$.

II. 1. Shew that $9^2 \times 2^7$ and $6^2 \times 12^2 \times 2$ represent the same number.

2. What is meant by "cancelling" between two numbers?

3. Write down all the common measures of
30, 48, and 96.

4. Write down five common multiples of 4 and 6.

5. What are the rules for ascertaining whether numbers are multiples of 3 and of 8? Express in simple factors,
71104.

III. 1. Find the value of

$$3 \times 2^2 - 4 \times 3^2 + 2 \times 5^2 - 3 \times 2^3.$$

2. Write down the quotient when
$3^3 \times 8 \times 7^2$ is divided by $3 \times 2^3 \times 7$.

3. Explain the terms
Greatest Common Measure, Least Common Multiple.
Write down both the G. C. M. and the L. C. M. of 8 and 12.

4. Find the G. C. M. of 96, 144, 240, 360.

5. Find the L. C. M. of 96, 144; also of 240, 360; also of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

IV. 1. Write down three fractions each equal to 9.

2. Simplify $1\frac{1}{2}$ of $2\frac{1}{3}$ of $3\frac{1}{4} \div 4\frac{1}{5}$.

3. Simplify $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{4} - 4\frac{1}{5}$.

4. Simplify $\dfrac{1\frac{1}{3} \text{ of } 3\frac{1}{4}}{4\frac{1}{3}} \div \dfrac{1\frac{1}{3} \text{ of } 3\frac{1}{5}}{5\frac{1}{3}}$.

5. Find the value of $2\frac{5}{6}$ of half-a-crown.

V. 1. Divide $8\frac{3}{4}$ by $4\frac{3}{8}$.

2. Bring to Lowest Terms, $\frac{360}{520}$.

3. Simplify $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$.

4. Simplify $(\frac{1}{2} \div \frac{1}{3})$ of $(\frac{1}{4} \div \frac{1}{5})$ and $\frac{1}{2} \div \frac{1}{3}$ of $(\frac{1}{4} \div \frac{1}{5})$.

5. Bring $\frac{2}{9}$ of 3 yds. to the fraction of 4ft. 9in.

VI. 1. To what are the following fractions respectively equal,

$\frac{130}{10}$, $\frac{13}{1}$, $\frac{13}{2}$, $\frac{13}{12}$, $\frac{13}{13}$, $\frac{13}{130}$, $\frac{13}{1313}$?

2. How many *tenths* are there in $13\frac{3}{10}$? how many in $13\frac{1}{2}$?

3. Simplify $\dfrac{\frac{1}{2}+\frac{2}{3}-\frac{3}{4}}{\frac{1}{2}\times\frac{2}{3}\div\frac{3}{4}}$.

4. Simplify $\dfrac{13\frac{1}{3}-3\frac{1}{13}}{13\frac{1}{3}+3\frac{1}{13}} \div \dfrac{4\frac{3}{4}\text{ of }2\frac{1}{3}}{4\frac{3}{4}\div 2\frac{1}{3}}$.

5. Simplify

$\frac{3}{5}$ of £1 2s. 6d. + $\frac{4}{15}$ of £2 2s. 6d. − $\frac{4}{25}$ of £3 15.

VII. 1. What are expressed by

$.3;\ 1.3;\ .13;\ .03;\ .0\dot{1}3$?

2. Add together 1.3, 1.03, 1.003, $.000\dot{3}$; and express the sum as a mixed number.

3. Simplify $1.7\times.9\div.\dot{5}\dot{1}$.

4. Find the value of 1.075 of £5.

5. Express $2\frac{1}{2}$ inches as the decimal of a foot.

VIII. 1. Express as decimals *without dividing the Numerators by the Denominators,*

$\frac{1}{2},\ \frac{1}{3},\ \frac{3}{50},\ \frac{4}{33},\ \frac{7}{45}$.

2. Simplify $.13-3.1+4.0\dot{2}$.

3. Divide $.\dot{1}2\dot{3}$ by 41, by .3, and by $.00\dot{3}$.

4. Find the value of $2.0\dot{3}$ of 5 half-crowns.

5. Simplify 21.12 of an hour − 211.2 of 6 minutes.

IX. 1. Find the value of

$\frac{1}{2}$ of $.39+\frac{1}{3}$ of $.4\dot{9}+\frac{1}{4}$ of $.\dot{5}\dot{9}-.\dot{0}6\dot{9}$.

2. Divide .0051 by .017 and multiply the quotient by 30.

3. Simplify $\dfrac{.\dot{3}\times.\dot{4}}{.3\div.4}$.

4. Reduce .363 of 1 mile to furlongs, yards, and feet.

5. Reduce 1 furlong, 1 yard, 2 feet to the decimal of 1 mile.

X. 1. Divide 1000 by 99, using factors; and state the reason for the method used for finding the true remainder.

2. Write down some examples of incorrect cancelling, and simplify, with as little working as possible,

$$\frac{3\frac{2}{3} \times \{7 \div 2\frac{1}{4}\}}{14 \times \{3\frac{2}{3} \div 1\frac{1}{8}\}}$$

3. Simplify $\dfrac{.\frac{1}{2} - .\frac{1}{3} + .\frac{1}{4}}{\frac{1}{.\dot{2}} - \frac{1}{.\dot{3}} + \frac{1}{.\dot{4}}}$

4. Subtract $.9\dot{1}$ of £9 from $.1\dot{9}$ of £90 0s. 2.5d.

5. Find the number of minutes in .3 of 3 days; also bring it to the decimal of .4 of 4 days.

XI. 1. Subtract from $.2\dot{1}\dot{3}$ its hundredth part, and hence bring it to a vulgar fraction.

2. What do the Numerator and Denominator of a fraction respectively express?

Shew by means of examples that

$\frac{1}{3} = \frac{3}{9}$ and that $\frac{30}{40} = \frac{3}{4}$.

3. Find a fraction with Numerator 20 which shall be double of one third of $1\frac{2}{3}$.

4. Find which is the greater

the difference between .45 and $.4\dot{5}$,

or, that between $.4\dot{5}$ and $.\dot{4}\dot{5}$.

5. By how many tenths of a second
does $3.7037\dot{3}$ of $\frac{5}{6}$ of an hour
exceed 4.629625 of $\frac{2}{3}$ of an hour?

XII. 1. Shew by means of an example
that $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{4}$.

2. Express in simple factors
7600, 4180, 3960,
and find their G. C. M.

3. What must be added to $8\frac{1}{3}$ to make $9\frac{1}{4}$,
and to 1.23 to make $1.2\dot{3}$?

4. By what must 3.33 of a mile be multiplied that it may be equal to 33.3 of a furlong?

5. Find $\frac{999}{1000}$ of $.2\dot{3}4\dot{5}$, and hence bring the decimal to a Vulgar Fraction.

ANSWERS TO THE EXAMINATION PAPERS.

IV. (2) $2\frac{17}{24}$; (3) $2\frac{53}{60}$; (4) $1\frac{1}{4}$;
(5) 7s. 1d.

V. (3) $\frac{13}{60}$; (4) $1\frac{7}{8}$, $1\frac{1}{5}$; (5) $\frac{8}{19}$.

VI. (3) $\frac{15}{16}$; (4) $\frac{45}{392}$; (5) £1 16s. 10d.

VII. (4) £5 7s. 6d.; (5) $.208\dot{3}$.

VIII. (4) £1 5s. 5d.

IX. (1) $.441\dot{5}8749\dot{6}$; (4) 2fur. 198yds. 2.64ft.;
(5) $.12594\dot{6}9$.

X. (3) $1\frac{1}{9}$; (4) £27 16s. 1d; (5.) .5625.

XI. (5) 1.

XII. (4) 79200.

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